FREQUENCY SENSITIVE
QFT WEIGHTING MATRIX

THESIS

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SELECTION OF A FREQUENCY SENSITIVE QFT WEIGHTING MATRIX USING THE METHOD OF SPECIFIED OUTPUTS

THESIS

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Preface

I would like to thank my thesis advisor and committee for their guidance and suggestions. The influence of Dr. C. H. Houpis, Dr. I. M. Horowitz, Lieutenant Colonel Z. H. Lewantowicz, and Major D. Gleason was a good and positive experience. I especially would like to thank Lieutenant Colonel Lewantowicz for helping me understand his Method of Specified Outputs. I would also like to thank my AFIT compadre Captain Paul Whalen and my roommate Captain Sesh Munipalli for their help in using LaTeX. Finally, I would like to thank my rock climbing buddies Steve Payson, Steve Parker, Paul Whalen, Dave Duvall, and Rich Walker, and my ASD/AFEF and ASD/ENFTC friends who helped me get the most out of the experience called Ohio.

William D. Phillips

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A ≡ State space model plant matrix.

a ≡ a numerator coefficient of the $\delta_{ij}(j\omega)$ transfer function.

B ≡ State space model control matrix.

b ≡ a denominator coefficient of the $\delta_{ij}(j\omega)$ transfer function.

C ≡ State space model output matrix.

$\underline{c}_{ij}$ ≡ the vector of coefficients that describe $\delta_{ij}(j\omega)$.

D ≡ State space model feedforward matrix.

det ≡ determinant of a matrix.

E ≡ Control surface effectiveness (a fraction of deflection capability).

F ≡ matrix of complex numbers.

$\hat{F}$ ≡ row reduced echelon form of F or modified Hermite normal form of F.

f ≡ the number of evaluations at different frequencies.

i ≡ a dummy variable used for counting.

j ≡ a dummy variable used for counting.

k ≡ a dummy variable used for counting.

l ≡ the number of system outputs.

$\hat{l}$ ≡ the number of system outputs desired to be controlled.

m ≡ the number of control inputs.

MIMO ≡ multiple-input multiple-output.

p ≡ roll rate.

$P(s)$ ≡ plant matrix.

$P_e(s)$ ≡ equivalent plant matrix.

P_{e_f} ≡ failed equivalent plant matrix.

$P_f(s)$ ≡ failed plant matrix.

P_o ≡ nominal plant.

q $\equiv$ degree of numerator polynomial of $\delta_{ij}(j\omega)$.

q $\equiv$ pitch rate.

QFT $\equiv$ Quantitative Feedback Theory.

r $\equiv$ degree of denominator polynomial of $\delta_{ij}(j\omega)$.

r $\equiv$ yaw rate.

s $\equiv$ Laplace transform variable.

SISO $\equiv$ single-input single-output.

^{T} $\equiv$ transpose of a matrix.

TED $\equiv$ Trailing edge down.

TEL $\equiv$ Trailing edge left.

TER $\equiv$ Trailing edge right.

TEU $\equiv$ Trailing edge up.

T_R $\equiv$ an equivalent roll mode time constant.

u $\equiv$ forward velocity.

$\underline{U}$ $\equiv$ vector of Laplace transformed control inputs.

u_{lf} $\equiv$ left flaperon deflection.

u_r $\equiv$ rudder deflection.

u_{rf} $\equiv$ right flaperon deflection.

$\underline{V}$ $\equiv$ vector of Laplace transformed desired outputs.

$\underline{X}$ $\equiv$ vector of Laplace transformed state variables.

$\underline{Y}$ $\equiv$ vector of Laplace transformed system outputs.

Z $\equiv$ a matrix composed of selected elements of $\hat{F}$.

α $\equiv$ angle of attack.

β $\equiv$ sideslip angle.

$\Delta(s)$ $\equiv$ frequency sensitive weighting matrix.

$\delta_{ij}(s)$ $\equiv$ element ij of $\Delta(s)$.

δ_{nij} $\equiv$ numerator of $\delta_{ij}(s)$.

$j\omega \equiv$ complex frequency.

$\phi \equiv$ bank angle.

$\theta \equiv$ pitch angle.

$\omega \equiv$ frequency.

$^{-1} \equiv$ inverse of a matrix.

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Abstract

Use of Quantitative Feedback Theory (QFT) on a multiple-input multiple-output control system requires certain mathematical properties of the plant matrix of system transfer functions $P(s)$. In general, the plant matrix $P(s)$ does not possess the necessary or desired mathematical properties for the QFT design to proceed. A frequency sensitive weighting matrix $\Delta(s)$ is used to transform the plant matrix $P(s)$ into an equivalent plant matrix $P_e(s)$ that does satisfy QFT requirements. In matrix notation, the relationship between the equivalent plant, plant, and weighting matrices is $P_e(s) = P(s)\Delta(s)$. This thesis identifies the necessary and desired characteristics of the equivalent plant $P_e(s)$ for the QFT design process, explains the use of the Method of Specified Outputs which generates the frequency sensitive weighting matrix $\Delta(s)$, and calculates several $\Delta(s)$ matrices for a 3-input 2-output lateral-directional model of the AFTI/F-16 aircraft. For several control system failures, the mathematical structure of the failed equivalent plant matrices $P_{e_f}(s)$ is examined for compliance with QFT requirements. A weighting matrix $\Delta(s)$ is found that produces acceptable equivalent plant matrices for failures of down to 0.01% of available control surface deflections. The use of the software packages MATRIX_X and MACSYMA is explained as applied to the Method of Specified Outputs.

SELECTION OF A FREQUENCY SENSITIVE QFT WEIGHTING MATRIX USING THE METHOD OF SPECIFIED OUTPUTS

I. Introduction

Quantitative Feedback Theory (QFT) is a multiple-input multiple-output (MIMO) control system design technique that accounts for uncertainties inherent in the system's plant state space model [Hou87:1-1]. Use of Quantitative Feedback Theory requires certain mathematical properties of the plant's state space model. The system's plant model does not always have the necessary properties for the QFT design to proceed. The goal of this thesis is to provide the tools to modify the original plant matrix into a form that can be used in the QFT design process.

For a MIMO system, the plant's dynamics can be described as a matrix of transfer functions, the plant matrix $P(s)$, that relate the m control inputs $\underline{U}(s)$ to the l system outputs $\underline{Y}(s)$. See Figure 1.1. Because of approximations and measurement limitations in determining the plant model, there exists an envelope of uncertainty for the plant's parameters at each design condition. The QFT technique, which requires a square equivalent plant matrix $P_e(s)$, attempts to determine a closed-loop control system that incorporates a cascade compensator $G(s)$ and an input filter $F(s)$ that provides acceptable system performance throughout the range of plant uncertainties. See Figure 1.2 for the QFT compensated MIMO system [Hou87:3-7]. Note that in Figure 1.2, the block labeled $P_e(s)$ is the square equivalent plant matrix which is explained below.

As stated earlier, the plant's state space model is not always in a form that allows the QFT design process to proceed. QFT design requires a square plant

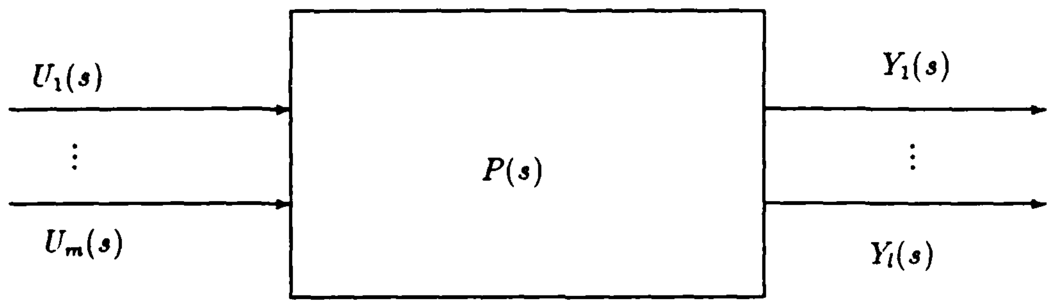

Figure 1.1. Uncompensated Plant

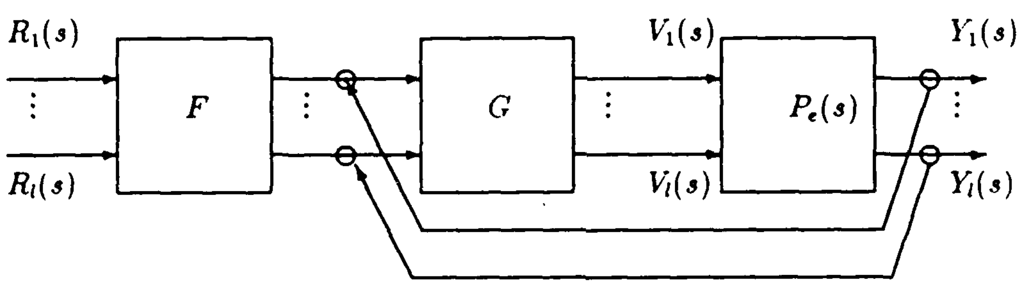

Figure 1.2. QFT MIMO Control Structure

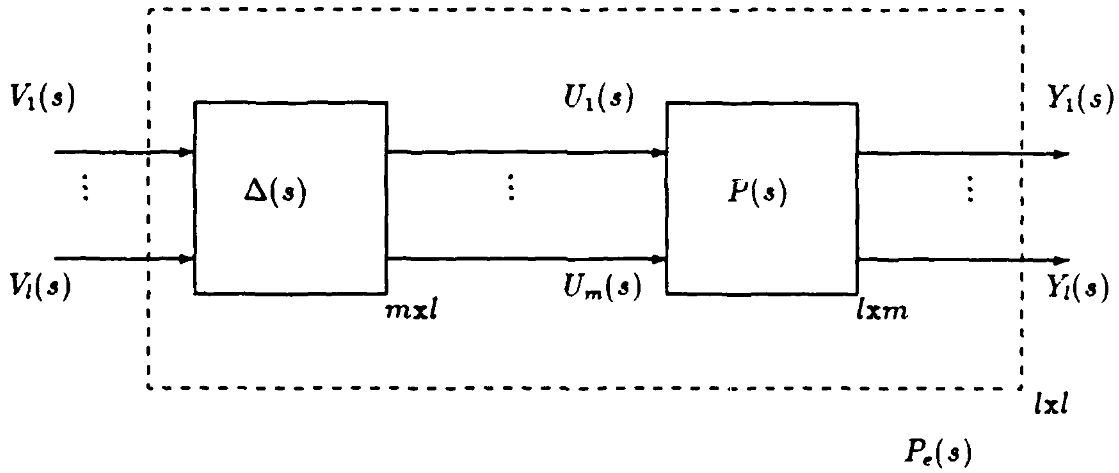

Figure 1.3. Equivalent Plant

matrix. If the number of control inputs m exceeds the number of system outputs l, then an "optimization" problem exists for exploiting the available control degrees of freedom [Hor88]. A frequency sensitive weighting matrix $\Delta(s)$ is used to modify the original plant matrix in order to exploit the available degrees of control freedom. Specifically, the system's plant matrix $P(s)$ is postmultiplied by a weighting matrix $\Delta(s)$ to form an equivalent plant matrix $P_e(s)$. See Eq. (1.1) and Figure 1.3. Note that $\underline{V}(s)$ is the vector of desired outputs.

$$P_e(s) = P(s)\Delta(s) \tag{1.1}$$

This thesis focuses on selection of a frequency sensitive weighting matrix $\Delta(s)$ that provides the necessary mathematical structure to the plant matrix $P(s)$ so that QFT multiple-input multiple-output design can proceed. The Method of Specified Outputs is used to calculate $\Delta(s)$. Note that the Method of Specified Outputs technique can be applied to other multiple-input multiple-output control theories that use a weighting matrix to transform the original non-square plant

matrix into a square equivalent plant matrix. The "squaring-down" matrix used by Porter is an example [PM88:34-55].

1.1 Background

Past QFT designs have used a weighting matrix Δ of constant, real elements δ_{ij} in order to transform the original plant matrix $P(s)$ into an equivalent plant matrix $P_e(s)$ suitable for the QFT design technique. There have been various methods used to determine the magnitude and sign for each Δ element δ_{ij}. The first attempts in determining δ_{ij} used elements of 0, +1, and -1. Engineering judgment was used in selecting the δ_{ij}'s. One technique was to analyze Bode plots of the original plant's transfer functions to determine which control inputs had the largest influence upon the system outputs. Later QFT designs incorporated non-unity δ_{ij}'s but they were still real-valued constants. [Ham87:22]

Last year, Lieutenants Hamilton and Adams began using frequency dependent $\delta_{ij}(s)$ elements in their respective digital QFT designs. However, LT Hamilton's choice of the structure of one particular $\delta_{ij}(s)$ resulted in unacceptable output and he was forced to abandon the frequency sensitive $\delta_{ij}(s)$ and go back to constant valued δ_{ij}'s. [Ham87:39]. Also, LT Adams optioned to use constant δ_{ij}'s in his design. [Ada88]

1.2 The Problem

In an attempt to provide QFT designers an equivalent plant matrix $P_e(s)$ that has both the necessary and desired structure over the entire range of original plant $P(s)$ uncertainty, a frequency sensitive weighting matrix $\Delta(s)$ is sought. Additionally, the use of $\Delta(s)$ on the original plant $P(s)$ is actually an open-loop, cascade compensation of the original open-loop control system as seen in Figure 1.1. Thus the use of a weighting matrix can improve open-loop performance in addition to preparing the system for QFT design. The primary purpose of this effort is

to find a $\Delta(s)$ that provides an equivalent plant $P_e(s)$ that satisfies the QFT constraints.

1.3 Scope

To achieve a solution to the above problem, the Method of Specified Outputs is used. The methodology is examined and a lateral-directional model of the AFTI/F-16 aircraft is used as an example of the method. This analysis is performed only in the continuous Laplace variable s-domain. Digital considerations are not addressed.

1.4 Assumptions

It is assumed that the QFT designer has a linear state space model of the system in the form

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{1.2}$$

$$\underline{y} = C\underline{x} + D\underline{u} \tag{1.3}$$

Where $\underline{x}$ = state variables, $\underline{u}$ = control inputs, $\underline{y}$ = system outputs, and A, B, C, and D are matrices of real, constant elements. The A, B, C, and D matrices are only valid at specific design conditions. As the system's environment changes, so do the A, B, C, and D matrices. For example, if the system modeled is an aircraft, then a flight condition of Mach 0.2 at 1,000 feet altitude produces much different A, B, C, and D matrices than a flight condition of Mach 1.5 at 30,000 feet altitude.

The actual system being modeled determines the assumptions needed to transform the system's nonlinear, coupled, differential equations of motion into linear, first order differential equations in the state space domain. For example, aircraft state space model assumptions are different than those made for a ship, a robot, or a missile. For the Method of Specified Outputs of Chapter III, it is assumed the frequency response of the original plant matrix $P(s)$ is not corrupted with noise. Thus, the coefficients of $\delta_{ij}(s)$ is uniquely determined.

For the AFTI/F-16 aircraft control example of Chapter V, the following assumptions are made:

1. The aircraft equations of motion are linearized about the nominal equilibrium (trim) condition of Mach 0.9 at 20,000 feet.
2. A small perturbation model is valid.
3. Input commands and aircraft response do not violate the linear model assumptions.
4. The mass of the aircraft remains constant during the commanded maneuvers.
5. Aircraft thrust is constant.
6. The aircraft is a rigid body.
7. The earth is an inertial reference frame.
8. The atmosphere is fixed to the earth.
9. The dimensional stability derivatives for flaperon inputs can be halved to provide derivatives for individual, thus independent, surface contributions.
10. The dimensional stability derivatives are constant for the given flight condition.
11. Control system failures can be described by scaling columns of the control matrix B of the state space model Eq. (1.2). Scaling refers to a positive scalar less than one multiplying an appropriate column of the B matrix. This assumption means the aircraft's stability derivatives taken with respect to the states, which are contained in the A and C matrices of Eqs. (1.2) and (1.3), do not appreciably change when a control surface is failed as either frozen in an arbitrary position or is unable to achieve maximum deflection. With this assumption, many control system failures can be modeled.
12. Laplace transforms exists for all signals and all system elements.

See [Bar83] and [Sch86].

1.5 Materials and Equipment

The analysis is performed using MATRIX_X software on a VAX 11-780 digital computer. MATRIX_X is a programmable matrix calculator with graphics and it has extensive control system analysis capabilities. [MAT86]

Also, MACSYMA software on a VAX 11-785 digital computer is used to perform symbolic (as opposed to numerical) matrix multiplication, inversion, and determinants. [VAX85]

1.6 Presentation

Chapter II describes the QFT constraints on the equivalent plant matrix. These in turn constrain the degrees of freedom for choosing the weighting matrix $\Delta(s)$ that transforms the plant matrix $P(s)$ into the equivalent plant matrix $P_e(s)$.

Chapter III describes the Method of Specified Outputs. This method is used to determine the frequency sensitive weighting matrix $\Delta(s)$.

Chapter IV describes the selection criteria for the QFT weighting matrix based upon the QFT constraints of Chapter II and the Method of Specified Outputs of Chapter III.

Chapter V explores an example of the Method of Specified Outputs technique using a lateral-directional model of the AFTI/F-16. Analysis and results are presented.

Chapter VI summarizes conclusions of this work and recommendations for further investigation.

Appendix A provides examples of MATRIX_X command files useful for the numerical aspect of the Method of Specified Outputs.

Appendix B provides examples of MACSYMA commands useful for the symbolic matrix manipulation aspect of the Method of Specified Outputs.

II. QFT and the Equivalent Plant Matrix

2.1 Introduction

Use of Quantitative Feedback Theory requires the plant matrix of transfer functions to have certain mathematical properties. In general, the plant does not possess the necessary and desired properties for the QFT design technique to be applied. Therefore, the plant's state space model is modified into the desired form by open-loop weighting matrix compensation. See Figure 1.3.

In the past, researchers used a single weighting matrix of constant, real elements to perform the necessary mathematical modification of the plant matrix [Ham87:22]. They had some success. However, this thesis is an attempt to investigate a technique to find a single, frequency sensitive weighting matrix $\Delta(s)$ which provides an equivalent plant transfer function matrix $P_e(s)$ with the necessary and desired mathematical properties over the entire range of plant uncertainties.

2.2 Plant Matrix $P(s)$

The plant matrix $P(s)$ is the transfer function matrix that relates all system inputs to all system outputs. The system is modeled as a set of linear, first order, vector differential equations

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{2.1}$$

$$\underline{y} = C\underline{x} + D\underline{u} \tag{2.2}$$

where $\underline{x}$ is the vector of state variables, $\underline{u}$ is the vector of control inputs, $\underline{y}$ is the vector of system outputs, and the A, B, C, and D matrices contain real constants. The elements of the state, input, and output vectors are functions of time. Let m be the number of control inputs and l be the number of system outputs. The vector $\underline{u}$ has dimension mx1 and the vector $\underline{y}$ has dimension lx1. Taking the Laplace

transform of these equations and assuming zero initial conditions gives

$$\underline{X} = (sI - A)^{-1}B\underline{U} \tag{2.3}$$

$$\underline{Y} = C\underline{X} + D\underline{U} \tag{2.4}$$

where $\underline{X}$, $\underline{U}$, and $\underline{Y}$ are the Laplace transforms of $\underline{x}(t)$, $\underline{u}(t)$ and $\underline{y}(t)$ respectively. Solving for Y(s) in terms of A,B,C,D and U(s) gives

$$\underline{Y} = \left[C(sI - A)^{-1}B + D\right]\underline{U} \tag{2.5}$$

Let the plant matrix $P(s)$ be defined as

$$P(s) \equiv C(sI - A)^{-1}B + D \tag{2.6}$$

Note that P(s) is an uncompensated, open-loop control system with m inputs and l outputs as seen in Figure 1.1. The plant $P(s)$ is a lxm matrix of transfer functions.

The plant matrix $P(s)$ embodies the multiple-input multiple-output (MIMO) system model as seen in Figure 1.1 and in Eq. (2.5). A different plant matrix $P(s)$ exists for each different environment or system failure encountered. For example, if the system being modeled is an aircraft, a different $P(s)$ exists for each flight condition and for each control system failure [i.e. for any possible combination of the plant parameters contained in A, B, C, or D of Eq. (2.6)].

For the entire range of $P(s)$'s at a given design condition, there exists mathematical properties that are either necessary or very desirable for the MIMO QFT design technique. In general, $P(s)$ does not possess the necessary or desired properties. To solve this problem, a weighting matrix $\Delta(s)$ is used to transform $P(s)$ in to an equivalent plant matrix $P_e(s)$ that does satisfy the QFT requirements. It is hoped that all $P(s)$'s within the region of plant uncertainty can be made to satisfy the QFT requirements with the use of a single $\Delta(s)$.

2.3 Equivalent Plant $P_e(s)$ and Weighting Matrix $\Delta(s)$

The equivalent plant is obtained by post-multiplying the original nominal plant matrix by a weighting matrix $\Delta(s)$.

$$P_e(s) = P(s)\Delta(s) \tag{2.7}$$

The basis for Eq. (2.7) can be seen from the following observations of Figure 1.3: The original, uncompensated, input-output relationship is given by

$$\underline{Y} = P(s)\underline{U} \tag{2.8}$$

By introducing a weighting matrix $\Delta(s)$ cascaded with the original plant matrix as in Figure 1.3, the input-output relation for the $\Delta(s)$ matrix is

$$\underline{U} = \Delta(s)\underline{V} \tag{2.9}$$

where the vector $\underline{V}$ is the Laplace transform of the desired output vector $\underline{v}(t)$. Substituting Eq. (2.9) into Eq. (2.8) gives the equivalent plant input-output relationship of Figure 1.3

$$\underline{Y} = P(s)\Delta(s)\underline{V} \tag{2.10}$$

The equivalent plant $P_e(s)$ is defined as

$$P_e(s) \equiv P(s)\Delta(s) \tag{2.11}$$

Thus the overall input-output relationship Eq. (2.10) can be written

$$\underline{Y} = P_e(s)\underline{V} \tag{2.12}$$

Recall that $\underline{V}$ is the vector of Laplace transformed desired outputs and $\underline{Y}$ is the vector of Laplace transformed actual outputs. Therefore, defining $P_e(s)$ such that $P_e(s) = P(s)\Delta(s)$ provides the designer a mechanism to modify the system plant model into a form useful for QFT design. The $\Delta(s)$ matrix is the mechanism to achieve the useful form.

Note that the use of $\Delta(s)$ is essentially an open-loop matrix compensation of the original MIMO system. Improvement in system performance is expected with proper choice of the weighting matrix $\Delta(s)$ in addition to preparing the system model for use by the Quantitative Feedback Theory's technique [Lew88]. Of course, the number one priority of the weighting matrix $\Delta(s)$ is for QFT purposes.

2.4 Characteristics of the Equivalent Plant $P_e(s)$

2.4.1 QFT Requirements on the Equivalent Plant This section deals with the required mathematical structure of the equivalent plant matrix $P_e(s)$. The justification for these requirements come from the Multiple-Input Single-Output (MISO) Equivalent Method used in MIMO QFT designs [Hou87:3-9]. Multiple-input multiple-output QFT designers reduce their MIMO systems to a set of equivalent MISO systems. To use the MISO equivalent models, the following conditions must be met.

Necessary Condition 1 The equivalent plant matrix $P_e(s)$ must be non-singular for both the nominal plant and for the range of plant uncertainties [Hou87:3-15]. Thus $P_e(s)$ must be controllable [Hor88]. The equivalent plant matrix must be square and the determinant of $P_e(s)$ must be non-zero for any possible combinations of plant parameters [Hou87:3-15].

A square equivalent plant matrix requirement means that the number of control inputs equals the number of system outputs. In any multiple-input multiple-output system with m inputs, there are at most m outputs which can be independently controlled. [Hou87:3-2]. Thus the dimensions of $P_e(s)$ will be $l x l$ where $l \leq m$.

It is easy to find an example where the original plant $P(s)$ is not square. Consider a control problem where there are more control inputs available than outputs that the designer wishes to control (i.e. $P(s)$ has more columns than rows). For example, if the system being modeled is an aircraft, there are often

more control surfaces than aircraft output variables that the designer is interested in controlling. For example, in the lateral-directional model of an aircraft, one may desire to control sideslip and roll rate but the available independent control surfaces are left and right flaperons, left and right differential horizontal tail, and rudder. In this case, there are 5 control inputs and 2 system outputs and the plant matrix $P(s)$ is 2x5. In order for $P_e(s)$ to be 2x2, then $\Delta(s)$ must be 5x2.

Necessary Condition 2, Only for the 2x2 Case For an equivalent plant matrix with 2 inputs and 2 outputs

$$P_e(s) = \begin{bmatrix} p_{e_{11}} & p_{e_{12}} \\ p_{e_{21}} & p_{e_{22}} \end{bmatrix} \tag{2.13}$$

the diagonal dominance condition for the 2x2 equivalent plant states that as $\omega \rightarrow \infty$ then

$$\left| p_{e_{11}} p_{e_{22}} \right| > \left| p_{e_{12}} p_{e_{21}} \right| \tag{2.14}$$

See [Hou87:3-17]

Necessary Condition 3, Only for the 3x3 Case The effective plant matrix for the 3x3 case is

$$P_e(s) = \begin{bmatrix} p_{e_{11}} & p_{e_{12}} & p_{e_{13}} \\ p_{e_{21}} & p_{e_{22}} & p_{e_{23}} \\ p_{e_{31}} & p_{e_{32}} & p_{e_{33}} \end{bmatrix} \tag{2.15}$$

and its inverse is

$$P(s)^{-1} = \begin{bmatrix} p^{*}_{e_{11}} & p^{*}_{e_{12}} & p^{*}_{e_{13}} \\ p^{*}_{e_{21}} & p^{*}_{e_{22}} & p^{*}_{e_{23}} \\ p^{*}_{e_{31}} & p^{*}_{e_{32}} & p^{*}_{e_{33}} \end{bmatrix} \tag{2.16}$$

If the QFT designer is using the Single-Loop Equivalents technique of [Hou87:3-19] on a 3x3 plant, then the following condition must hold: as $\omega \rightarrow \infty$

$$\left|p^{*}_{e_{11}} p^{*}_{e_{22}} p^{*}_{e_{33}}\right| \geq \left|p^{*}_{e_{12}}\right| \left(\left|p^{*}_{e_{21}} p^{*}_{e_{33}}\right| + \left|p^{*}_{e_{23}} p^{*}_{e_{31}}\right|\right) + \left|p^{*}_{e_{13}}\right| \left(\left|p^{*}_{e_{22}} p^{*}_{e_{31}}\right| + \left|p^{*}_{e_{21}} p^{*}_{e_{32}}\right|\right) + \left|p^{*}_{e_{11}} p^{*}_{e_{23}} p^{*}_{e_{32}}\right| \tag{2.17}$$

If the QFT designer is using the improved method instead of the Single-Loop Equivalents technique, then the following condition must hold: as $\omega \rightarrow \infty$ then $det\left[\frac{P}{P_0}\right]$ must not change sign where P_0 is the nominal plant [Hor88]. See [Hor79] for higher order plants.

2.4.2 <u>*Desired Characteristics of the System Model*</u> This section deals with the desired characteristics of the equivalent plant matrix $P_e(s)$.

<u>Desired Characteristic 1</u> The determinant of the equivalent plant matrix $P_e(s)$ should preferably be minimum phase for the nominal equivalent plant, and for all possible combinations of control system failures. Thus it is desired that $\det[P_e(s)]$ be minimum phase for the entire range of plant uncertainties. That is, since the elements of $P_e(s)$ are quotients of polynomials in the Laplace variable s, then the zeros of the determinant of $P_e(s)$, $\det[P_e(s)]$, should lie in the left-half s-plane for both the healthy and failed plants. If zeros of $\det[P_e(s)]$ do lie in the right-half s-plane, then they should preferably be far into the r.h.p. so as to minimize their effect [Hor88].

A sufficient condition for $\det[P_e(s)]$ to be made minimum phase is that at least one of the lxl minors of $P(s)$ be minimum phase over the entire range of plant parameter variations [Hou87:5-19] and [Sch86:11]. This conclusion comes from the Binet-Cauchy formula [Lan69].

As noted previously, this thesis does not consider the digital QFT design technique. The digital QFT designer should realize that sampled systems are inherently non-minimum phase [Hou87:E-8].

<u>Desired Characteristic 2</u> The equivalent plant matrix should be as diagonal as possible [Hor88]. If $P_e(s)$ is diagonal, then no cross-coupling occurs between control inputs and system outputs. That is, one input effects only one output. Generally, however, all failed equivalent plants $P_{e_f(s)}$ are not diagonal.

<u>Desired Characteristic 3</u> For a stable original plant $P(s)$, the response of the

equivalent plant $P_e(s)$ should be reasonable and stable for reasonable control inputs for the entire range of plant parameter variations. For stability, the poles of the equivalent plant matrix (i.e. roots of the denominator polynomial for each plant element) should lie in the left-half s-plane. A reasonable response means that the outputs are not excessive provided that the inputs are not unusually large and that both the input and output limits are not exceeded. The reasonableness of a response is a judgement made by the QFT designer. The designer must consider the maximum and minimum input control limits and output response limits of the system. For example, if the system being modeled is an aircraft and if a step rudder input of 5 degrees causes the sideslip angle to be greater than 95 degrees, then that scenario would fail the reasonability test. For the same aircraft, if an aileron deflection of 200 degrees were required to cause a roll rate of 1 degree per second, that scenario would also fail the reasonability test.

A reasonable response implies that the elements of P_e must be realizable. That is, each transfer function of $P_e(s)$ must have at least as many poles as zeros.

The remainder of this thesis focuses on the selection of a frequency sensitive weighting matrix $\Delta(s)$ by use of the Method of Specified Outputs. For the example of Chapter 5, the resulting equivalent plant $P_e(s)$ is in the correct form for the QFT design to proceed.

III. *The Method of Specified Outputs*

3.1 Introduction

The Method of Specified Outputs allows calculation of a frequency sensitive weighting matrix $\Delta(s)$ for use with the Quantitative Feedback Theory design procedure. The Method of Specified Outputs was developed by Lieutenant Colonel Zdzilaw H. Lewantowicz [Ham87]. As noted in Chapter II, the method can be thought of as an open-loop compensation of the system. The goal is to find a *single* weighting matrix $\Delta(s)$ which provides an acceptable $P_e(s)$ for not only the healthy $P(s)$ but also for the entire range of failed plant matrices $P_f(s)$.

3.2 The Method of Specified Outputs

The Method of Specified Outputs is based upon Eq. (3.1)

$$P_e(s) = P(s)\Delta(s) \tag{3.1}$$

The key to the method is specifying the equivalent plant matrix $P_e(s)$ that satisfies the QFT constraints described in Chapter II. If $P_e(s)$ and $P(s)$ are given, then $\Delta(s)$ can be determined from linear algebra transformations discussed below.

CASE 1: If the original plant has more control inputs than system outputs, then $P(s)$ has more columns than rows and $\Delta(s)$ can be defined as the minimum norm solution to Eq. (3.1) provided that $P(s)$ has linearly independent rows. The minimum norm solution to Eq. (3.1) is

$$\Delta(s) = P(s)^T[P(s)P(s)^T]^{-1}P_e(s) \tag{3.2}$$

CASE 2: If the original plant $P(s)$ has the same number of inputs as outputs and if $P(s)^{-1}$ exists but $P(s)$ does not meet the QFT constraints of Chapter II, then Δ is given as

$$\Delta(s) = P(s)^{-1}P_e(s) \tag{3.3}$$

CASE 3: If the original plant $P(s)$ has fewer control inputs than system outputs, some of the outputs will be uncontrollable. In this case, an exact solution for $\Delta(s)$ is not possible but a least squares solution is given by Eq. (3.4) provided that $P(s)$ has linearly independent columns.

$$\Delta(s) = [P(s)^T P(s)]^{-1} P(s)^T P_e(s) \tag{3.4}$$

3.2.1 The Transformation of $P(s)$ into $P_e(s)$ The $\Delta(s)$ matrix transforms the $P(s)$ matrix into the $P_e(s)$ matrix. The columns of the $\Delta(s)$ matrix "steer" linear combinations of the columns of the $P(s)$ matrix into the columns of the $P_e(s)$ matrix [Lew88]. For the minimum norm and exact solutions of cases 1 and 2 above, the transformation of $P(s)$ into $P_e(s)$ space is exact. For the least squares solution of case 3 above, the P matrix can not be transformed into P_e space but the error of the projection of P into P_e space can be minimized. The following section describes Eq. (3.2) thru Eq. (3.4). The selection of $P_e(s)$ is discussed in Chapter IV.

3.2.1.1 The Minimum Norm Solution For the case where $P(s)$ has more columns than rows, the minimum norm solution for $\Delta(s)$ is given by Eq. (3.5). When $P(s)$ has more columns than rows, this is equivalent to saying that there are more control inputs $\underline{u}$ than system outputs $\underline{y}$. This is an underconstrained matrix algebra problem and there are infinitely many exact solutions for $\Delta(s)$, provided that the rank of $P(s)P(s)^T$ is as large as possible. The minimum norm solution for $\Delta(s)$ is

$$\Delta(s) = P(s)^T [P(s)P(s)^T]^{-1} P_e(s) \tag{3.5}$$

The expression $P(s)^T(P(s)P(s)^T)^{-1}$ is called the right inverse of $P(s)$ and is thus a pseudoinverse of $P(s)$ [Gel74:19]. If $P(s)$ is post-multiplied by its right inverse, then the identity matrix results. If the minimum norm solution for Δ is substituted into Eq. (3.1), then the following identity results

$$P_e(s) = P(s)P(s)^T[P(s)P(s)^T]^{-1}P_e(s) \tag{3.6}$$

or

$$P_e(s) = P_e(s) \tag{3.7}$$

The necessary condition for the minimum norm solution for $\Delta(s)$ to exist is if

$$rank(PP^T) = l \tag{3.8}$$

where l = row dimension of P.

Eq. (3.8) says $P(s)$ must have linearly independent rows and that $rank(PP^T)$ is as large as possible. If Eq. (3.8) holds, then a unique $(PP^T)^{-1}$ exists, the right inverse of $P(s)$ exists, and the minimum norm solution for $\Delta(s)$ exists [Str88:96]. Note that (PP^T) is an lxl square symmetric matrix.

The minimum norm solution is only one of infinitely many possible solutions for the underconstrained case where $P(s)$ has more columns than rows. The term "minimum norm" comes from the fact that the right inverse (i.e. $P^T[PP^T]^{-1}$) minimizes the norm of each column of Δ(s). That is, each column of $\Delta(s)$ is a solution vector of minimum length [Gel74:19].

This minimum norm solution for $\Delta(s)$ is used in the example of Chapter V.

3.2.1.2 <u>*The P^{-1} Solution*</u> For the case where $P(s)$ has the same number of columns and rows, and if $rank(P) = l = m$, then a unique $P(s)^{-1}$ exists. Only one solution for $\Delta(s)$ exists and is given by

$$\Delta(s) = P(s)^{-1}P_e(s) \tag{3.9}$$

The QFT designer can use Eq. (3.9) if $P(s)$ does not possess the desired plant matrix properties described in Chapter II. By properly specifying P_e and using Eq. (3.9), the desired equivalent plant matrix properties can be achieved for at least the healthy plant.

3.2.1.3 *Least Squares Solution* For the case where $P(s)$ has more rows than columns and $P(s)$ has linearly independent columns, then the least squares solution for $\Delta(s)$ is

$$\Delta(s) = [P(s)^T P(s)]^{-1} P(s)^T P_e(s) \tag{3.10}$$

For Eq. (3.10) to have a valid solution, then the following condition must be met

$$rank(P^T P) = m \tag{3.11}$$

where m = column dimension of P. Eq. (3.11) says P must have linearly independent columns and that $rank(P^T P)$ is as large as possible [Str88:15].

When $P(s)$ has more rows than columns, this is equivalent to saying there are more system outputs $\underline{y}$ than control inputs $\underline{u}$. Obviously, the excess of outputs $\underline{y}$ over the inputs $\underline{u}$ can never be controlled. This is an overconstrained matrix algebra problem and no exact solution exits. Eq. (3.10) guarantees the error between $P_e(s)$ and $P(s)\Delta(s)$ is minimized in the least squared sense.

Note that $P^T P$ is mxm square and symmetric. If the condition of Eq. (3.11) holds, then $(P^T P)^{-1}$ exists, the left inverse of P exists, and Eq. (3.10) applies. The left inverse of P is defined as $(P^T P)^{-1} P^T$ and is a pseudoinverse of P. If P is premultiplied by its left inverse, the identity matrix results, i.e.,

$$(P^T P)^{-1} P^T P = I \tag{3.12}$$

Substituting Eq. (3.10) into Eq. (3.1) gives

$$P_{e,apprx} = P(P^T P)^{-1} P^T P_e \tag{3.13}$$

Note the left hand side of Eq. (3.13) is denoted $P_{e,apprx}$ because it is not exactly the specified P_e since no exact solution exists. The matrix $P(P^TP)^{-1}P^T$ projects P_e onto the column space of P and $P_{e,apprx}$ is that projection [Str88:158].

3.2.2 Numerical Determination of $\Delta(s)$ The above section implies that the calculation of $\Delta(s)$ is performed symbolically. That is, for the minimum norm solution $\Delta = P^T(PP^T)^{-1}P_e$, there is a matrix product calculation PP^T, a matrix inversion $(PP^T)^{-1}$, and two more matrix products $P^T(PP^T)^{-1}$ and $P^T(PP^T)^{-1}P_e$ before $\Delta(s)$ is determined. Recalling that each element of $P(s)$ is a transfer function in s (i.e. the quotient of two polynomials in the Laplace variable s) it is clear that the symbolic calculation of $\Delta(s)$ is time consuming if performed by hand calculation. It is difficult for even a 1x2 plant with a denominator polynomial of degree 3 and the numerator polynomials of degree 2. To speed up the calculation of $\Delta(s)$, the QFT designer can use a symbolic manipulator like MACSYMA [VAX85] or the QFT designer can determine $\Delta(s)$ numerically at selected complex frequencies $s_k = j\omega_k$ and determine the denominator and numerator polynomial coefficients using a systems identification scheme [Ham87]. Note that numerically, $\Delta(j\omega_k)$ is calculated exactly because both $P(j\omega_k)$ and $P_e(j\omega_k)$ are known exactly by evaluating $P(s)$ and $P_e(s)$ at $s = j\omega_k$.

Both conceptual approaches for determining $\Delta(s)$ are examined for the example of Chapter V. It is not possible to sufficiently simplify the symbolic MACSYMA expression for $\Delta(s)$ into a manageable form. Therefore, the numerical approach is chosen for the example of Chapter V. Once the transfer functions of $\Delta(s)$ are determined numerically using MATRIX$_X$, then MACSYMA is used to perform the matrix product $P_e(s) = P(s)\Delta(s)$.

Determining $\Delta(s)$ numerically is essentially a mathematical systems identification problem. Two such system identification techniques are considered. One technique was devised by Lieutenant Colonel Z. Lewantowicz and the other by Lieu-

tenant Colonel W. Baker both of the Air Force Institute of Technology. The scheme devised by LTC Baker seems well suited for this MIMO application but there was insufficient time to investigate this technique thoroughly. See reference [Bak80] for a MIMO systems identification technique and references [Lev59] and [Pay70] for a SISO systems identification technique. The remainder of this section discusses the determination of $\Delta(s)$ using LTC Lewantowicz's systems identification technique.

Evaluate the appropriate $\Delta(s)$ solution of Section 3.2.1 at a number of complex frequencies $s = j\omega$. The minimum number of necessary frequency evaluations f is discussed below. Consider an individual element of the numerically computed weighting matrix $\delta_{ij}(j\omega)$. Each element is a complex number which is the evaluation of a transfer function at a specific frequency. The element's transfer function is of the form

$$\delta_{ij}(j\omega) = \frac{a_q(j\omega)^q + a_{q-1}(j\omega)^{q-1} + \cdots + a_1(j\omega) + a_0}{b_r(j\omega)^r + b_{r-1}(j\omega)^{r-1} + \cdots + b_1(j\omega) + b_0} \tag{3.14}$$

where q is the degree of the numerator polynomial and r is the degree of the denominator polynomial. The goal is to determine the real-valued constant coefficients $a_q, a_{q-1}, \cdots, a_1, a_0, b_r, b_{r-1}, \cdots, b_1, b_0$ for each element δ_{ij} of the weighting matrix $\Delta(s)$. One reason for calculating these coefficients is the desire to determine the time histories of the outputs of $\Delta(s)$. For the transfer function to be realizable (i.e. proper), then $q \leq r$. Note that the outputs of $\Delta(s)$ are the inputs to the original plant $P(s)$. See Eq. (2.9). The method used to calculate $\Delta(s)$ imposes no path constraints on the time histories of the outputs of Δ and thus should be checked for reasonableness.

The following technique determines the coefficients for each $\delta_{ij}(s)$. Once all $\delta_{ij}(s)$'s have been determined, a common denominator polynomial for the $\Delta(s)$ matrix can be found. Again note that when $\delta_{ij}(j\omega)$ is evaluated at a particular complex frequency $j\omega_k$, then $\delta_{ij}(j\omega_k)$ is a complex number. In matrix notation,

the scalar Eq. (3.14) becomes

$$\begin{bmatrix} (j\omega)^q & (j\omega)^{q-1} & \cdots & 1 \end{bmatrix} \begin{bmatrix} a_q \\ a_{q-1} \\ \vdots \\ a_0 \end{bmatrix} = \delta_{ij}(j\omega) \begin{bmatrix} (j\omega)^r & (j\omega)^{r-1} & \cdots & 1 \end{bmatrix} \begin{bmatrix} b_r \\ b_{r-1} \\ \vdots \\ b_0 \end{bmatrix} \tag{3.15}$$

Further manipulation of Eq. (3.15) yields a matrix equation that evaluates to the scalar zero

$$\begin{bmatrix} \begin{bmatrix} (j\omega)^q & (j\omega)^{q-1} & \cdots & (j\omega) & 1 \end{bmatrix} & -\delta_{ij}(j\omega) \begin{bmatrix} (j\omega)^r & (j\omega)^{r-1} & \cdots & (j\omega) & 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} a_q \\ a_{q-1} \\ \vdots \\ a_1 \\ a_0 \\ b_r \\ b_{r-1} \\ \vdots \\ b_1 \\ b_0 \end{bmatrix} = 0 \tag{3.16}$$

Define $\underset{\sim}{c}_{ij}$ as the vector of coefficients that define the transfer function $\delta_{ij}(j\omega)$

$$\underset{\sim}{c}_{ij} \equiv \begin{bmatrix} a_q \\ a_{q-1} \\ \vdots \\ a_1 \\ a_0 \\ b_r \\ b_{r-1} \\ \vdots \\ b_1 \\ b_0 \end{bmatrix} \tag{3.17}$$

Note that $\underset{\sim}{c}_{ij}$ has the dimension $[q+r+2]$x1.

The objective of the system identification technique is to identify each element of $\underset{\sim}{c}_{ij}$. This can be done because $\delta(j\omega)_{ij}$ can be calculated exactly at any frequency $j\omega_k$. To use LTC Lewantowicz's technique, evaluate Eq. (3.16) at f frequencies where $f \geq (q+r+2)$

$$\begin{bmatrix} \begin{bmatrix} (j\omega_1)^q & (j\omega_1)^{q-1} & \cdots & (j\omega_1) & 1 \end{bmatrix} & -\delta(j\omega_1)\begin{bmatrix} (j\omega_1)^r & \cdots & 1 \end{bmatrix} \\ \begin{bmatrix} (j\omega_2)^q & (j\omega_2)^{q-1} & \cdots & (j\omega_2) & 1 \end{bmatrix} & -\delta(j\omega_2)\begin{bmatrix} (j\omega_2)^r & \cdots & 1 \end{bmatrix} \\ \vdots & \vdots \\ \begin{bmatrix} (j\omega_f)^q & (j\omega_f)^{q-1} & \cdots & (j\omega_f) & 1 \end{bmatrix} & -\delta(j\omega_f)\begin{bmatrix} (j\omega_f)^r & \cdots & 1 \end{bmatrix} \end{bmatrix} \begin{bmatrix} a_q \\ a_{q-1} \\ \vdots \\ a_1 \\ a_0 \\ b_r \\ \vdots \\ b_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3.18}$$

Note that Eq. (3.18) has the form $F\underset{\sim}{c}_{ij} = \underset{\sim}{0}$. Let F be the rectangular fx$[q+r+2]$ matrix of complex numbers that premultiplies $\underset{\sim}{c}_{ij}$. Therefore, $\underset{\sim}{c}_{ij}$ lies in the nullspace of F where F must be singular for a nullspace to exist. Note that

if F is non-singular, then the QFT designer should evaluate the numerical solution to $\Delta(s)$ at more distinct frequencies so that F is singular and a nullspace exists.

The vector $\underline{c}_{ij}$ defines only one transfer function of the weighting matrix $\Delta(s)$. Since $\Delta(s)$ has dimension mxl, then the procedure to find $\underline{c}_{ij}$ must be repeated $m \times l$ times. There are two ways of determining $\underline{c}_{ij}$; the first is to transform F into modified Hermite normal form. The second way is to perform a singular value decomposition on F.

Modified Hermite Normal Form Technique. Let F have at least as many rows as columns. Let $\hat{F}$ be the modified Hermite normal form of F. The modified Hermite normal form of a matrix is upper triangular (i.e. all elements below the main diagonal are zero), has either ones or zeros on the main diagonal, and any column with unity on the diagonal is otherwise zero [Rei83:330-331]. Note that a matrix in modified Hermite normal form is sometimes referred to as row reduced echelon form [Str88:77]. Replace any zero on the main diagonal of $\hat{F}$ with -1. Let Z be a matrix composed of the first $q + r + 2$ elements of any column of $\hat{F}$ that has a -1 on the main diagonal. The columns of Z span the nullspace of F and therefore any column of Z could be chosen to be the vector of coefficients $\underline{c}_{ij}$. However, care should be taken when selecting which column of Z should be chosen as $\underline{c}_{ij}$. F is a complex matrix and, in general, a set of complex vectors lie in the nullspace of F. However, in order for the transfer function δ_{ij} to have physical meaning, the elements of $\underline{c}_{ij}$ must be real. Therefore, chose the column of Z which has the negligible imaginary components and use the real components as $\underline{c}_{ij}$. To check that the real-valued $\underline{c}_{ij}$ does indeed lie in the nullspace of F, evaluate the matrix product $F\underline{c}_{ij}$. This product should be an fx1 vector of elements with each element's magnitude less than about 10^{-10}. Another useful check is to numerically evaluate the transfer function defined by $\underline{c}_{ij}$ and compare those complex numbers with the appropriate elements of the numerically determined $\Delta(s)$ solution of Section 3.2.1.

MATRIX_X has a row reduced echelon routine called RREF(matrix) [MAT86]. See Appendix A.

Singular Value Decomposition Technique. Any $f\text{x}[q+r+2]$ matrix F can be factored into

$$F = Q_1 \Sigma\, Q_2 \tag{3.19}$$

where the ξ singular values of F fill the first ξ places on the main diagonal of the $f\text{x}[q+r+2]$ dimensioned Σ matrix, the columns of the $f\text{x}f$ dimensioned Q_1 matrix are the eigenvectors of FF^T, and the columns of the $[q+r+2]\text{x}[q+r+2]$ dimensioned Q_2 matrix are the eigenvectors of F^TF. The last $[q+r+2]-\xi$ columns of Q_2 lie in the nullspace of F [Str88:443].

Again, any real vector that lies in the nullspace of F can be used as the vector of coefficients $\underline{c}_{ij}$ of the transfer function δ_{ij}. As before, the real part of one of the $n-r$ columns of Q_2 can be used for $\underline{c}_{ij}$.

MATRIX_X has a singular value decomposition routine called SVD(matrix) [MAT86].

As stated previously, once $\underline{c}_{ij}$ has been selected, it is important to check the accuracy of the approximation of $\delta_{ij}(s)$. Since $\Delta(s)$ can be calculated directly at any frequency using, for example, the minimum norm solution, it is important to compare those values against the calculation of $\delta_{ij}(s)$ using the coefficients of $\underline{c}_{ij}$.

IV. *Selection Criteria for the QFT Weighting Matrix $\Delta(s)$*

Once $P(s)$ and $P_e(s)$ are known, $\Delta(s)$ can be calculated using the Method of Specified Outputs technique of Chapter III. Therefore, the key to determining $\Delta(s)$ is really the proper choice of P_e. An equivalent title to this chapter could be *Selection Criteria for the QFT Equivalent Plant Matrix $P_e(s)$.*

Selecting an acceptable $P_e(s)$ can be straightforward. Each element of the $P_e(s)$ matrix is a transfer function that relates the actual system outputs $\underline{Y}$ to the desired (or commanded) inputs $\underline{V}$ ($\underline{V}$ is also the equivalent plant's input command vector). See Figure 1.3. The transfer function is a "tracking" function or a "rejecting" function depending on the QFT designer's wishes.

Let P_f designate a failed plant matrix. The P_f matrix is determined from Eq. (2.6) where the B matrix has been appropriately scaled in order to model a reduction in control effectiveness. Because a single weighting matrix $\Delta(s)$ is used to generate the equivalent plant matrix P_e throughout the entire range of plant failures P_f , two difficulties may arise. First, some system failures are catastrophic enough that the failed equivalent plant matrix $P_{e_f} = P_f\Delta$ no longer maintains the necessary or desired QFT properties. Second, the control inputs commanded by $\Delta(s)$ may be unreasonable in light of the control system's capabilities.

The following discussion contains guidance for selecting $P_e(s)$ which in turn determines $\Delta(s)$ when using the Method of Specified Outputs.

4.1 Requirements on $P_e(s)$

The following properties of $P_e(s)$ must exist if QFT is to be used.

4.1.1 Non-Singular $P_e(s)$ For $P_e(s)$ to be non-singular, it must be square and its determinant non-zero. That is, $P_e(s)$ must be controllable. The dimension

of the square P_e is $\hat{l}$ x $\hat{l}$ where $\hat{l}$ is the number of outputs that the designer wishes to control. Note that $\hat{l} \leq m$ where m is the number of independent control inputs.

4.1.2 Equivalent Plant Element Relationships Depending on the dimension of the equivalent plant matrix $P_e(s)$ and the particular QFT MIMO design technique used, a specific relationship must exist between the elements of the equivalent plant. In the 2x2 case, the diagonal dominance condition must hold. Similar conditions exist for higher dimensioned equivalent plant matrices.

4.1.3 Reasonable Response The designer should choose reasonable transfer functions for the elements of $P_e(s)$. If the output is to follow the input, then a first or second order tracking transfer function is a good initial choice.

If the output is desired to be zero for any input, then a first or second order disturbance rejection transfer function is a good initial choice for the elements on the main diagonal of $P_e(s)$ [Lew88]. Either disturbance rejection transfer functions or scalar zeros are recommended for off-diagonal elements of $P_e(s)$. See the cautionary note in section 4.2.2 on the use of scalar zeros as a disturbance rejection function on the main diagonal of $P_e(s)$.

To determine a reasonable choice for a transfer function as an element of $P_e(s)$, consider the control input hardware limits. Recall that this analysis assumes a linear system. If the control variables becomes saturated, the system becomes non-linear.

Another consideration in selection of elements of $P_e(s)$ is the original plant's frequency and time responses to the standard evaluation inputs in each of the controls u_i. For example, apply a step input of u_1 to the original plant P while holding all other inputs to zero. Observe both the time and frequency response of system outputs. Do not ask the equivalent plant $P_e(s)$ to provide a quantum jump in system performance when compared to the original plant $P(s)$.

For requirements on 2x2 and 3x3 equivalent plants, see section 2.4.1 of Chapter II.

Computer aided design tools such as MATRIX_X can only simulate the response of proper (i.e. realizable) transfer functions. Thus the degree of the numerator q must be less than or equal to the degree of the denominator r. The requirement for proper transfer functions apply not only for the elements of $P_e(s)$ but this limitation also applies to elements $\delta_{ij}(s)$ of the weighting matrix $\Delta(s)$. Note that the outputs of $\Delta(s)$ are the control vector $\underline{U}(s)$. In order to determine if the time histories $\underline{u}(t)$'s are reasonable for, say, step inputs to $\Delta(s)$, then the $\delta_{ij}(s)$'s must be realizable.

4.2 Desired Properties of $P_e(s)$

The following properties of $P_e(s)$ are desired in order that the QFT design may be simplified.

4.2.1 Minimum-Phase det$[P_e(s)]$ The designer should select the zeros of each transfer function of $P_e(s)$ so that det$[P_e(s)]$ are all minimum-phase (i.e. all zeros of the determinant lie in the left-half s-plane). The designer must check to see if all healthy and failed equivalent plant determinants are minimum-phase or if the right-half s-plane zeros of det$[P_{e_f}(s)]$ are so far into the r.h.p. that they can be ignored [Hor88].

4.2.2 Diagonal $P_e(s)$ A diagonal $P_e(s)$ is desirable so that non-diagonally related inputs and outputs are decoupled. A note of caution applies when using a diagonal P_e: a disturbance rejection transfer function on the main diagonal of a $P_e(s)$ can not be chosen to be the scalar zero. This causes det$[P_e]$ to be zero and thus violates the necessary condition 1 of section 2.4.1 and the MISO Equivalent method can not be applied [Hou87:3-9].

There is another advantage of using a diagonal $P_e(s)$. If the zeros of each transfer function of a diagonal $P_e(s)$ are chosen to be minimum-phase, then the determinant for the healthy plant will necessarily be minimum-phase.

V. *Example of the Method of Specified Outputs Technique*

To show the viability of the Method of Specified Outputs technique in providing an acceptable equivalent plant for QFT design, an AFTI/F-16 aircraft lateral-directional, 3-input/2-output model is examined. The model is from the identical flight condition considered in [Sch86].

5.1 *The Model*

The flight condition for the AFTI/F-16 is Mach 0.9 at 20,000 ft. The block diagram for this system is given in Figure 5.1. See Tables 5.1 and 5.2 for system inputs and outputs.

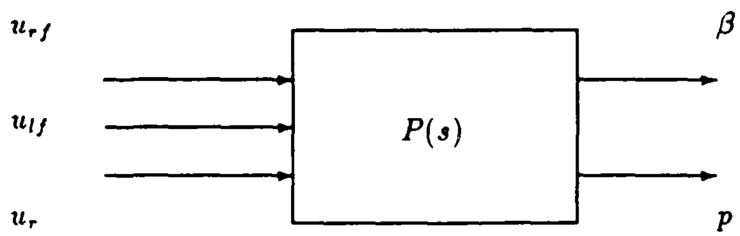

Figure 5.1. AFTI/F-16 Lateral-Directional 3-Input 2-Output Model

Table 5.1. Control Inputs and Units

Control Inputs	Description	Units
u_{rf}	right flaperon deflection	degrees
u_{lf}	left flaperon deflection	degrees
u_r	rudder deflection	degrees

The lateral-directional state space model is given in Eqs. (5.1) and (5.2)

$$\dot{\underline{x}} = A\underline{x} + B\underline{u} \tag{5.1}$$

Table 5.2. System Outputs and Units

System Outputs	Description	Units
β	sideslip angle	degrees
p	roll rate	degrees per second

$$\underline{y} = C\underline{x} + D\underline{u} \tag{5.2}$$

where $\underline{x}$ is the vector of 8 states, $\underline{u}$ is the vector of 3 control inputs, and $\underline{y}$ is the vector of 2 system outputs. See Eqs. (5.3) thru (5.5). The vector of state variables is

$$\underline{x} = \begin{bmatrix} \theta \\ u \\ \alpha \\ q \\ \phi \\ \beta \\ p \\ r \end{bmatrix} \tag{5.3}$$

See Table 5.3 for a description of the 8 states. The vector of control inputs is

Table 5.3. State Variables and Units

States	Description	Units
θ	pitch angle	degrees
u	forward velocity	feet per second
α	angle of attack	degrees
q	pitch rate	degrees per second
ϕ	bank angle	degrees
β	sideslip angle	degrees
p	roll rate	degrees per second
r	yaw rate	degrees per second

$$\underline{u} = \begin{bmatrix} u_{rf} \\ u_{lf} \\ u_r \end{bmatrix} \tag{5.4}$$

The vector of system outputs is

$$\underline{y} = \begin{bmatrix} \beta \\ p \end{bmatrix} \tag{5.5}$$

The state space model plant matrix is

$$A = [A_1 A_2] \tag{5.6}$$

where

$$A_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ -32.183 & 0.012075 & 38.2906 & -30.1376 \\ -0.00112 & -0.000022 & -1.48446 & 0.994789 \\ -0.000309 & -0.00013 & 4.27171 & -0.777221 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5.7}$$

and

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.03449 & -0.343554 & 0.032636 & -0.997556 \\ 0 & -55.2526 & -2.80004 & 0.145674 \\ 0 & 7.237 & -0.023184 & -0.36253 \end{bmatrix} \tag{5.8}$$

The control matrix is

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 1.158405 & 1.158405 & 0 \\ -0.122462 & -0.122462 & 0 \\ -3.236345 & -3.236345 & 0 \\ 0 & 0 & 0 \\ -0.0006855 & 0.0006855 & 0.037032 \\ -25.5251 & 25.5251 & 10.3955 \\ -0.62503 & 0.62503 & -5.8089 \end{bmatrix} \tag{5.9}$$

The output matrix is

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{5.10}$$

The feedforward matrix is

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{5.11}$$

The AFTI/F-16 flaperon deflection limits are ±20 degrees trailing edge up and down (TEU and TED). The rudder deflection limits are ±30 degrees trailing edge right and left (TER and TEL). The flaperon surface rate limit is 52 degrees per second and the rudder surface rate limit is 120 degrees per second [Bar83:Table 3].

5.2 The Plant Matrix, Both Healthy and Failed

The plant matrix relates all control inputs to system outputs. For the example, $P(s)$ has the dimensions 2x3, i.e. $l = 2$ which is the number of outputs and $m = 3$ which is the number of inputs. There are 6 transfer functions contained in $P(s)$.

$$P(s) = C(sI - A)^{-1}B \tag{5.12}$$

Define the healthy plant P as the evaluation of Eq. (5.12) for the A, B, C, and D matrices of Section 5.1. Define a failed plant P_f as the evaluation of Eq. (5.12) for the A, C, and D matrices of Section 5.1 and for a scaled B matrix that incorporates a reduction in control effectiveness. For this thesis, the healthy plant and the set of failed plants of Section 5.2.2 comprise the range of plant uncertainty that this thesis specifically addresses.

5.2.1 *The Healthy Plant* For the 8-state model, each element of P has a common denominator polynomial of degree 8. Upon inspection of each of $P(s)$'s 6 elements, there is considerable cancellation between the numerator zeros and the denominator poles. Therefore, the reduced order plant matrix is represented by 6 numerator polynomials of degree 3 and one denominator polynomial of degree 4. The time histories and Bode plots of the original, uncompensated plant are in Figures 5.2 thru 5.7.

Using the TFORM routine of MATRIX$_X$, the reduced order healthy plant matrix is

$$P(s) = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{bmatrix} \tag{5.13}$$

where

$$p_{11} = \frac{(-6.8550e-4)s^3 - (2.1170e-1)s^2 - (3.0527e-2)s - (3.2230e-1)}{dr} \tag{5.14}$$

$$p_{12} = -p_{11} \tag{5.15}$$

$$p_{13} = \frac{(3.7032e-2)s^3 + (6.2511)s^2 + (1.6957e+1)s + (1.0080e-1)}{dr} \tag{5.16}$$

$$p_{21} = \frac{(-2.5525e+1)s^3 - (1.8076e+1)s^2 - (2.2192e+2)s}{dr} \tag{5.17}$$

$$p_{22} = -p_{21} \tag{5.18}$$

$$p_{23} = \frac{(1.0396e+1)s^3 + (4.4478)s^2 - (2.4482e+2)s}{dr} \tag{5.19}$$

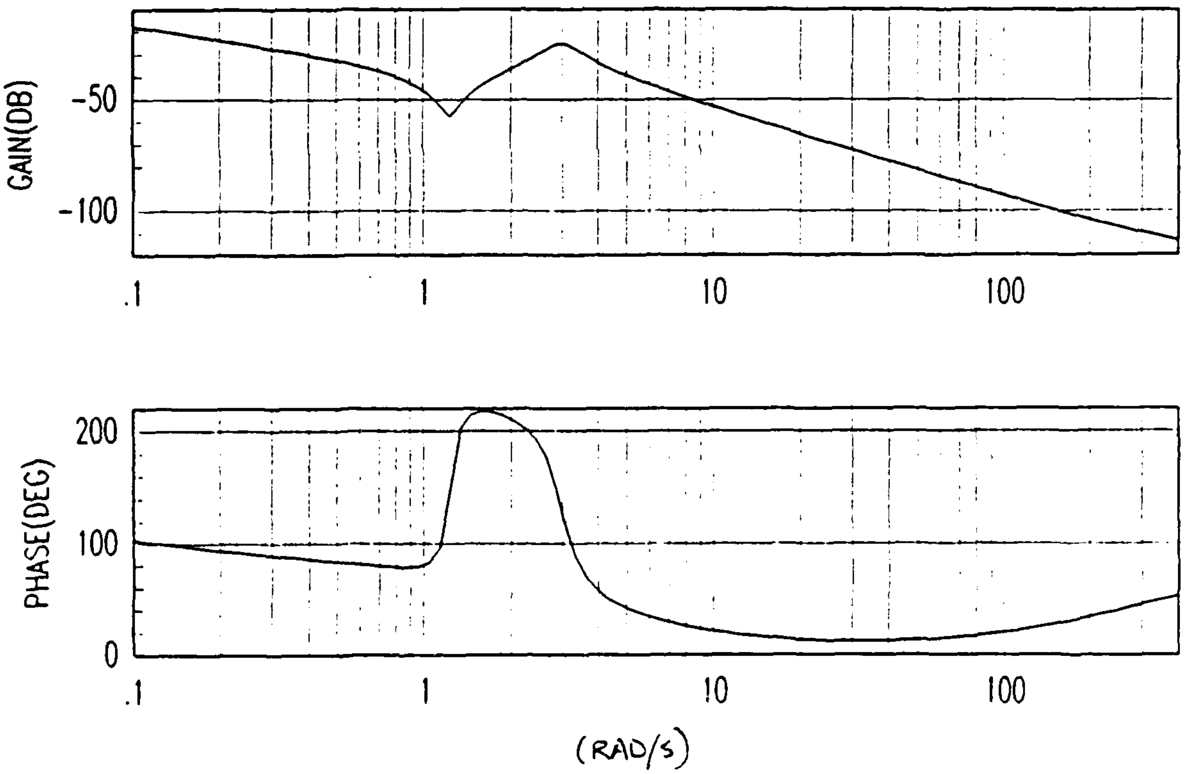

Figure 5.2. Original Plant, Response of Sideslip β to Right Flaperon u_{rf}

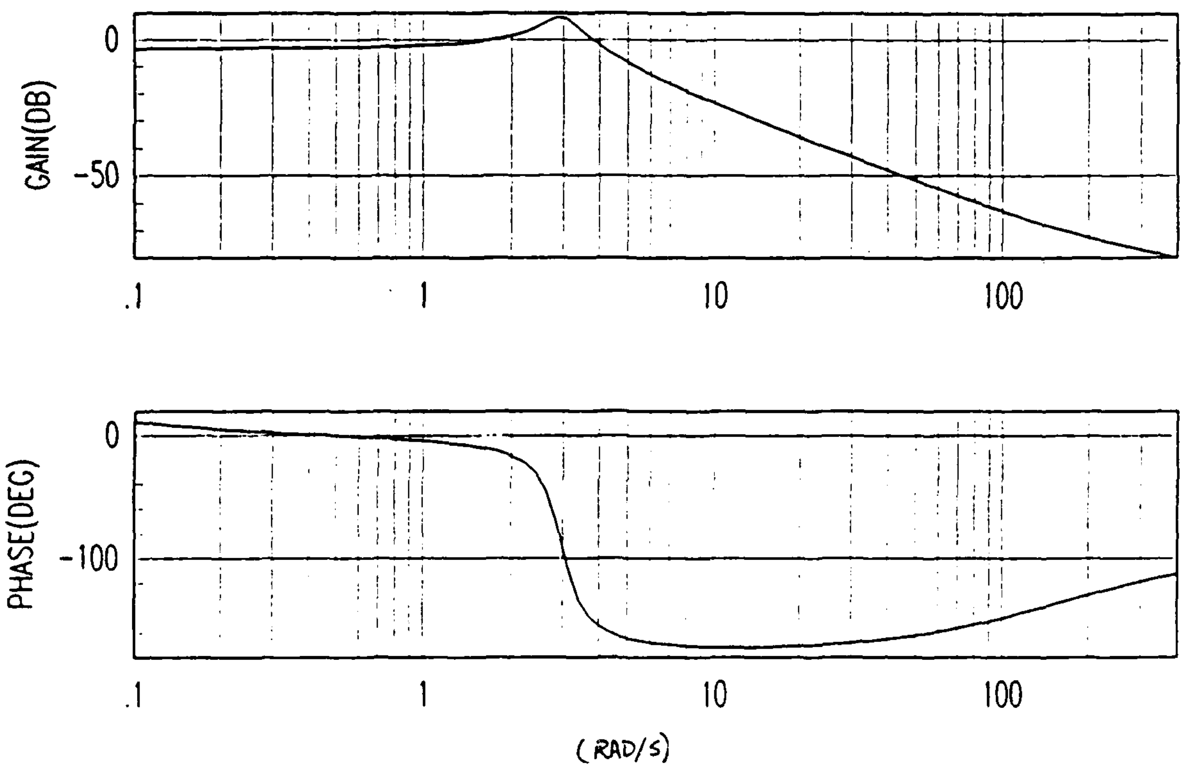

Figure 5.3. Original Plant, Response of Sideslip β to Rudder u_r

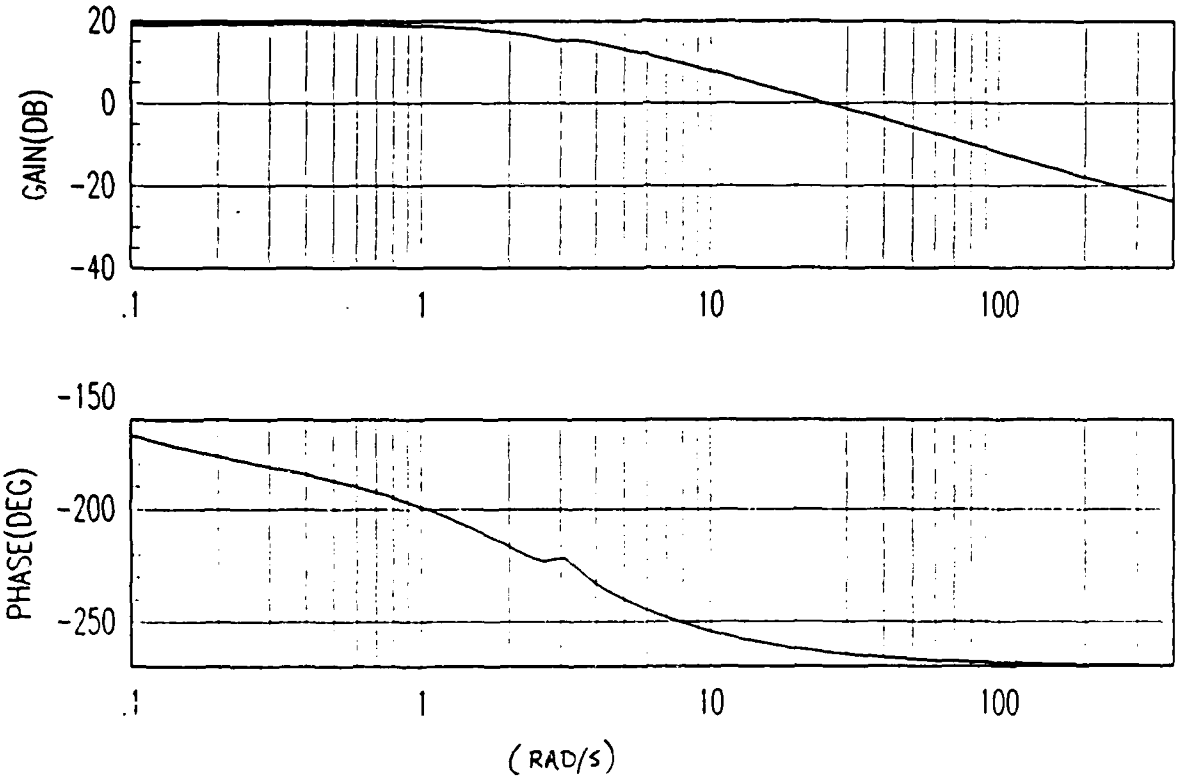

Figure 5.4. Original Plant, Response of Roll Rate p to Right Flaperon u_{rf}

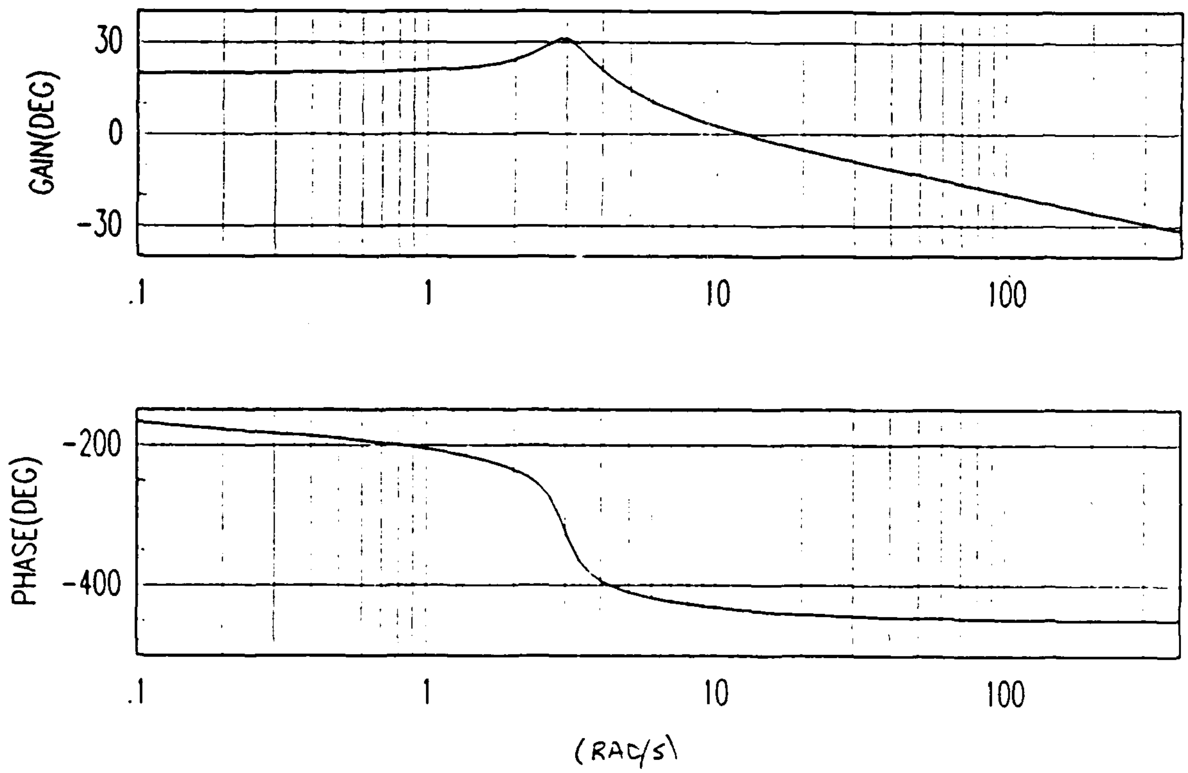

Figure 5.5. Original Plant, Response of Roll Rate p to Rudder u_r

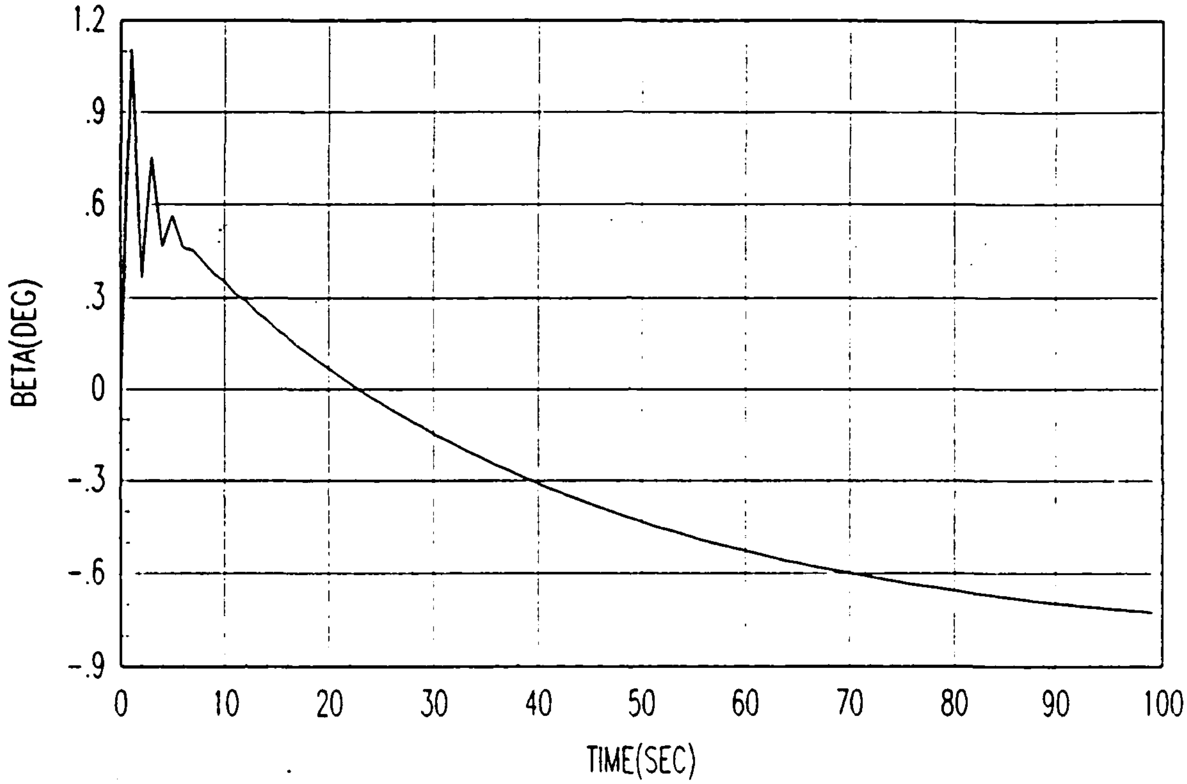

Figure 5.6. Original Plant, Sideslip β for a 1° Step Input to All Controls

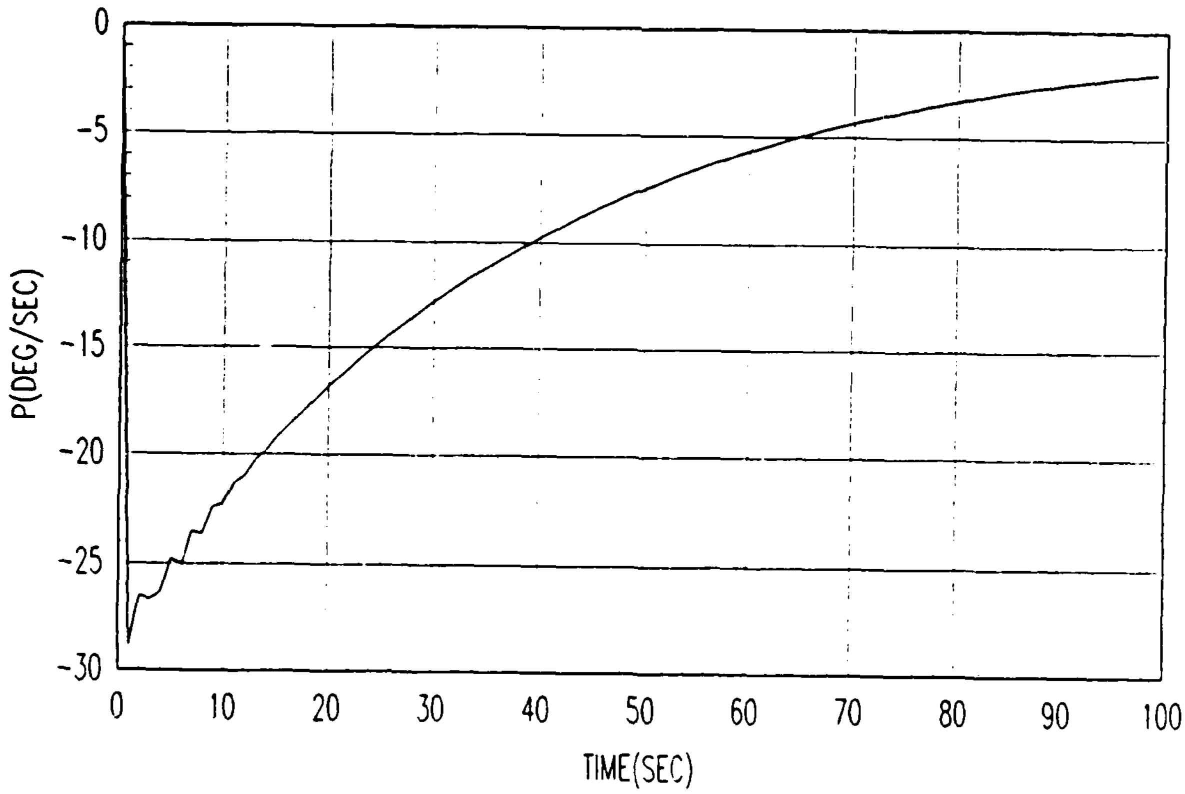

Figure 5.7. Original Plant, Roll Rate p for a 1° Step Input to All Controlls

and

$$dr = s^4 + 3.5061s^3 + (1.1238e+1)s^2 + (2.4367e+2)s + (6.5450e-1) \qquad (5.20)$$

Note the symmetry between p_{11} and p_{12} and between p_{21} and p_{22}. The p_{11} transfer function describes the β response to a right flaperon input and the p_{12} transfer function describes the β response to a left flaperon input. For a healthy aircraft, these responses are equal but opposite in sign.

The poles of the plant matrix (i.e. the roots of the denominator dr) all lie in the left-half s-plane and thus the original, uncompensated plant is stable.

$$polesp = \begin{bmatrix} -0.3910 + j2.9616 \\ -0.3910 - j2.9616 \\ -2.6969 \\ -0.02719 \end{bmatrix} \qquad (5.21)$$

5.2.2 <u>*The Failed Plants*</u> As stated in Section 5.2, the set containing the healthy plant and all the failed plants comprise the range of plant uncertainty for this thesis. The failures considered are listed in Table 5.4. An assumption is applied where plant failures can be modeled by reducing the effectiveness of the control inputs on the outputs by appropriately scaling the B matrix of Eq. (5.12). For example, if the right flaperon suffers a failure that reduces its range by 50 percent, then the healthy B matrix's first column is multiplied by 0.50. Denoting this failure as # 5, then $P_{failure5}$ is given by $C(sI - A)^{-1}B_{failure5}$. An equivalent description of a failure is illustrated by Eqs. 5.22 and 5.23 where E_i is the positive scalar less than or equal to 1 that describes the effectiveness of the i^{th} control surface.

$$\underline{Y}_{failure} = \begin{bmatrix} E_1P_{11} & E_2P_{12} & E_3P_{13} \\ E_1P_{21} & E_2P_{22} & E_3P_{23} \end{bmatrix} \begin{bmatrix} U_{rf} \\ U_{lf} \\ U_r \end{bmatrix} \qquad (5.22)$$

Table 5.4. Plant Failures

Failure No.	Surface Effectiveness		
	right flaperon	left flaperon	rudder
1	0%		
2			0%
3	0%		0%
4	0%	0%	
5	50%		
6			50%
7	50%		50%
8	50%	50%	
9	0%		50%
10	50%		0%
11	0.01%		
12			0.01%
13	0.01%		0.01%
14	0.01%	0.01%	
15	0.01%		50%
16	50%		0.01%
17	0.01%	0.01%	0.01%
Surfaces are 100% Effective Unless Noted			

or

$$\underline{Y}_{failure} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \end{bmatrix} \begin{bmatrix} E_1 U_{rf} \\ E_2 U_{lf} \\ E_3 U_r \end{bmatrix} \tag{5.23}$$

The failures in Table 5.4 are examined for this example.

Note that the stability of the failed plants is not influenced by scaling the B matrix. Since $P(s) = C(sI - A)^{-1}B$, then only the common denominator due to $(sI - A)^{-1}$ determines stability. Since this thesis assumes that the stability derivatives in the A matrix do not change as a result of failures, then the stability of the failed plants are identical to the healthy plant.

When using MATRIX$_X$, scaling a column of the B matrix is equivalent

to scaling the appropriate columns of NUM. NUM is the output matrix of the $[NUM, DEN] = TFORM(S, NS)$ routine. When using MACSYMA, scaling a column of the B matrix is equivalent to scaling a column of the plant matrix as in Eq. 5.23. See Appendices A and B.

5.3 Selection of Equivalent Plant

An equivalent plant matrix $P_e(s)$ is specified by the QFT designer based on the considerations of Chapters II and IV. For the AFTI/F-16 example, the equivalent plant is a 2x2 matrix of transfer functions relating the 2 inputs (v_1 and v_2) to the 2 outputs (sideslip angle β and roll rate p).

Recall the necessary and desired characteristics of an 2x2 equivalent plant for MIMO QFT purposes. The matrix $P_e(s)$ must be non-singular, $|p_{e_{11}}p_{e_{22}}| > |p_{e_{12}}p_{e_{21}}|$ as $\omega \rightarrow \infty$, $\det[P_e]$ should be minimum phase, P_e should preferably be diagonal, and the output of P_e should be stable and reasonable for a stable $P(s)$ and reasonable inputs.

From an open-loop performance perspective, the goal of the equivalent plant is to reject sideslip and follow a commanded roll rate. Thus $p_{e_{11}}$ is chosen to be a disturbance rejection transfer function, $p_{e_{22}}$ is chosen to be a tracking transfer function, and both $p_{e_{12}}$ and $p_{e_{21}}$ are chosen to be the scalar zero.

The healthy plant $P(s)$ and only one choice for $P_e(s)$ are the only quantities needed to determine a weighing matrix $\Delta(s)$ when using the Method of Specified Outputs. However, in order to compare the influence of the choice of $P_e(s)$ on the weighting matrix $\Delta(s)$, several $P_e(s)$'s are chosen. See Table 5.5. A common denominator of $(s + 10)^2$ is chosen for two reasons. First, for the $p_{e_{22}}(s)$ roll rate tracking function, this corresponds to a reasonable roll rate response for an AFTI/F-16 type aircraft. Define the roll mode time constant T_R as 63% of the time it takes to reach a steady state roll rate imparted by a step input [Ros82:458]. With this definition, T_R is 0.22 seconds as determined from Figure 5.9 which is

Table 5.5. Choices for $P_e(s)$

Choice	P_e
1	$\begin{bmatrix} \frac{(s+5)}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$
2	$\begin{bmatrix} \frac{(s+9)}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$
3	$\begin{bmatrix} \frac{(s+1)}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$
4	$\begin{bmatrix} \frac{(s+50)}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$
5	$\begin{bmatrix} \frac{0.01}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$

the time response to a unit step input to the transfer function $P_{e22} = \frac{100}{(s+10)^2}$. This compares favorably with $T_{R_{max}}$ of 1.0 seconds for a class IV aircraft during a category A flight phase with level 1 flying qualities. [Fly86:412].

Second, in order to compare the influence of the numerators of $p_{e_{11}}$ and $p_{e_{22}}$ on the phase of the determinant of the failed equivalent plants $\det[P_{e_f}]$, then a common denominator for $P(s)$ is a reasonable choice. Note that for all the equivalent plants in Table 5.5, the β rejection transfer function $p_{e_{11}}$ does not have a free s in the numerator and thus a steady state error exists for step inputs. In light of the desire to have no zeros of $\det[P_e]$ or $\det[P_{e_f}]$ in the right-half s plane or on the imaginary axis, it is reasonable to achieve β "rejection" by a gain much less than 1 in the $p_{e_{11}}$ transfer function and not by a free s in the numerator of $p_{e_{11}}$.

5.4 *Calculation of the Weighting Matrix $\Delta(s)$*

Recall that a single weighting matrix $\Delta(s)$ is used to transform the original plant $P(s)$ into the equivalent plant $P_e(s)$. This same weighting matrix $\Delta(s)$ attempts to transform a failed plant $P_f(s)$ into the equivalent plant. Of course,

$P_f(s)\Delta(s)$ is not equal $P(s)\Delta(s)$. However, it is hoped that $P_{e_f}(s) = P_f(s)\Delta(s)$ will maintain all the necessary and desirable QFT characteristics.

Since $P(s)$ has more inputs than outputs and since the rows of $P(s)$ are independent, then the minimum norm solution for $\Delta(s)$ applies. Recall the minimum norm solution $\Delta(s) = P(s)^T[P(s)P(s)^T]^{-1}P_e(s)$ where $P(s)$ is the healthy plant. Since $P(s)$ is 2x3 and since $P_e(s)$ is 2x2, then $\Delta(s)$ is 3x2.

In order to perform a simulation of the response of $\Delta(s)$ to any input, each transfer function $\delta_{ij}(s)$ of $\Delta(s)$ must be known. The outputs of $\Delta(s)$ are right flaperon deflection $U_{rf}(s)$, left flaperon deflection $U_{lf}(s)$, and rudder deflection $U_r(s)$. Recall that the coefficients of the numerator polynomial and denominator polynomial for δ_{ij} are represented by the vector $\underline{c}_{ij}$. Once all $\underline{c}_{ij}$'s are found, then a simulation of the frequency and time response of each δ_{ij} is conducted. Each time response is checked for reasonableness in light of the AFTI/F-16 hardware limits.

Each of the 6 transfer functions of $\Delta(s)$ are calculated using the numerical approach to the Method of Specified Outputs. Since $P(s)$ is known exactly [i.e. no measurements are needed to determine the frequency response of $P(s)$], then a moderate bandwidth of frequencies is used to calculate Δ(s). For each choice of equivalent plant, $\Delta(j\omega_k)$ is numerically evaluated at frequencies of ω_k from 0.5 to 15 radians per second at intervals of 0.5 radians per second. This range of frequencies is in the neighborhood of the bandwidth of the original plant's frequency response. Note that $f = 30$ is the number of frequencies at which $\Delta(j\omega)$ is evaluated. The QFT designer chooses the maximum degree of the numerator polynomial q and the degree of the denominator polynomial r for each δ_{ij}. The 30x$[q + r + 2]$ dimension F matrix is formed for δ_{ij} and $\underline{c}_{ij}$ is extracted from the columns of the modified Hermite normal form matrix $\hat{F}$. See Appendix A for an example of the MATRIX$_X$ command files used to numerically determine the weighting matrix transfer functions

5.5 *The Failed Equivalent Plants*

For this example, the Method of Specified Outputs guarantees that the product of the healthy plant and weighting matrix $P(s)\Delta(s)$ is exactly the specified healthy equivalent plant $P_e(s)$. All healthy plants are selected specifically to meet the necessary and desired QFT characteristics.

It is desired that all equivalent plants in the range of plant uncertainty meet the same standards as the healthy equivalent plant. However, if a failure is so severe as to cause the failed equivalent plant to be singular and thus uncontrollable or to cause $\det[P_{e_f}]$ to be non-minimum phase, then the QFT designer can either select another $P_e(s)$ and generate another $\Delta(s)$ or the designer can simply exclude that failure from the set of failed plants and proceed with the QFT design. Also, the QFT designer may be able to absorb the non-minimum phase case in the design if the r.h.p. zero is either very close to the origin or reasonably far into the r.h.p. [Hor88].

For this example problem, the determinant of the failed plants $\det[P_{e_f}]$ is symbolically evaluated using MACSYMA. The roots of the numerator polynomial of $\det[P_f\Delta]$ is examined for right-half s-plane zeros using both MACSYMA and MATRIX_X. See Appendix B for an example of the MACSYMA commands used.

5.6 *Results*

5.6.1 The $\Delta(s)$ for Each Equivalent Plant A frequency sensitive weighting matrix is determined by using the Method of Specified Outputs. For each choice of equivalent plant in Table 5.5, a weighting matrix $\Delta(s)$ is determined, the output of $\Delta(s)$ to step inputs in $V_1(s)$ and $V_2(s)$ is examined for reasonableness, the diagonal dominance condition is checked, and $\det[P_{e_f}]$ is examined for right-half s-plane zeros.

P_e Choice # 1

For $P_e = \begin{bmatrix} \frac{(s+5)}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$, the weighting matrix is

$$\Delta = \frac{\begin{bmatrix} \delta_{n11} & \delta_{n12} \\ \delta_{n21} & \delta_{n22} \\ \delta_{n31} & \delta_{n32} \end{bmatrix}}{(6.0627e-5)s^4 + (1.1213e-2)s^3 + (2.0606e-1)s^2 + s} \tag{5.24}$$

where

$$\delta_{n11} = (3.3591e-4)s^4 + (1.8233e-3)s^3 - (7.1924e-3)s^2 - (3.9555e-2)s \tag{5.25}$$

$$\delta_{n12} = (-1.1966e-4)s^3 - (2.0199e-2)s^2 - (5.4795e-2)s - (3.2570e-4) \tag{5.26}$$

$$\delta_{n21} = -\delta_{n11} \tag{5.27}$$

$$\delta_{n22} = -\delta_{n12} \tag{5.28}$$

$$\delta_{n31} = (1.6496e-3)s^4 + (9.4161e-3)s^3 + (2.0183e-2)s^2 + (7.1710e-2)s \tag{5.29}$$

$$\delta_{n32} = (-4.4301e-6)s^3 - (1.3681e-3)s^2 - (1.9729e-4)s - (2.0829e-3) \tag{5.30}$$

Figures 5.8 and 5.9 show the desired equivalent plant responses. Figures 5.10 and 5.11 show the time histories of the healthy right flaperon and healthy rudder to unit step inputs of 1° of both v_1 and v_2 into the weighting matrix $\Delta(s)$. The response of the healthy left flaperon is identical to the right flaperon. Since the model is assumed to be linear, then total right flaperon deflection u_{rf} is the sum of right flaperon deflection due to a 1° step input in v_1 and right flaperon deflection due to a 1° step input in v_2. The statement "the total response is the sum of the individual responses" also applies to the rudder deflection response. Recall the hardware limits on the AFTI/F-16 flaperon and rudder: flaperon deflection is 20° TEU and TED; rudder deflection is 30° TER and TEL. Based upon Figures 5.10 and 5.11 and the control surface limitations, the $\Delta(s)$ in Eq. 5.24 is not realistic. It is clear that inputs greater than 1° saturate the control u_r.

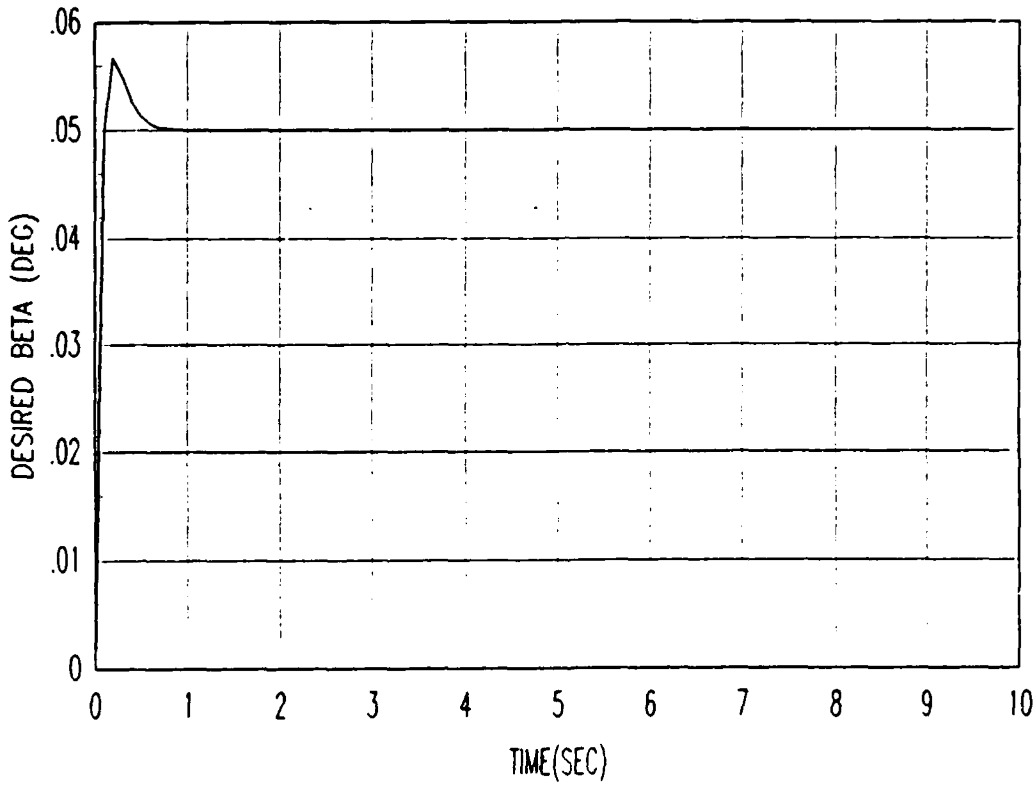

Figure 5.8. Desired Sideslip Unit Step Response for Choice #1

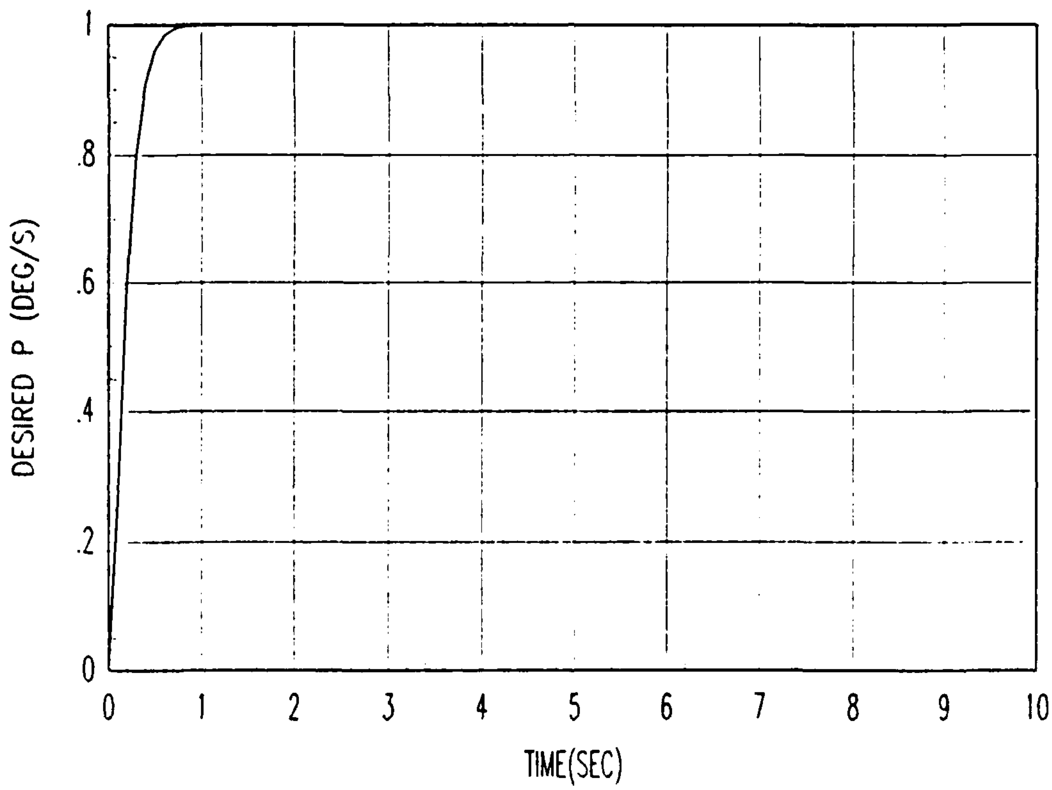

Figure 5.9. Desired Roll Rate Unit Step Response for All Choices

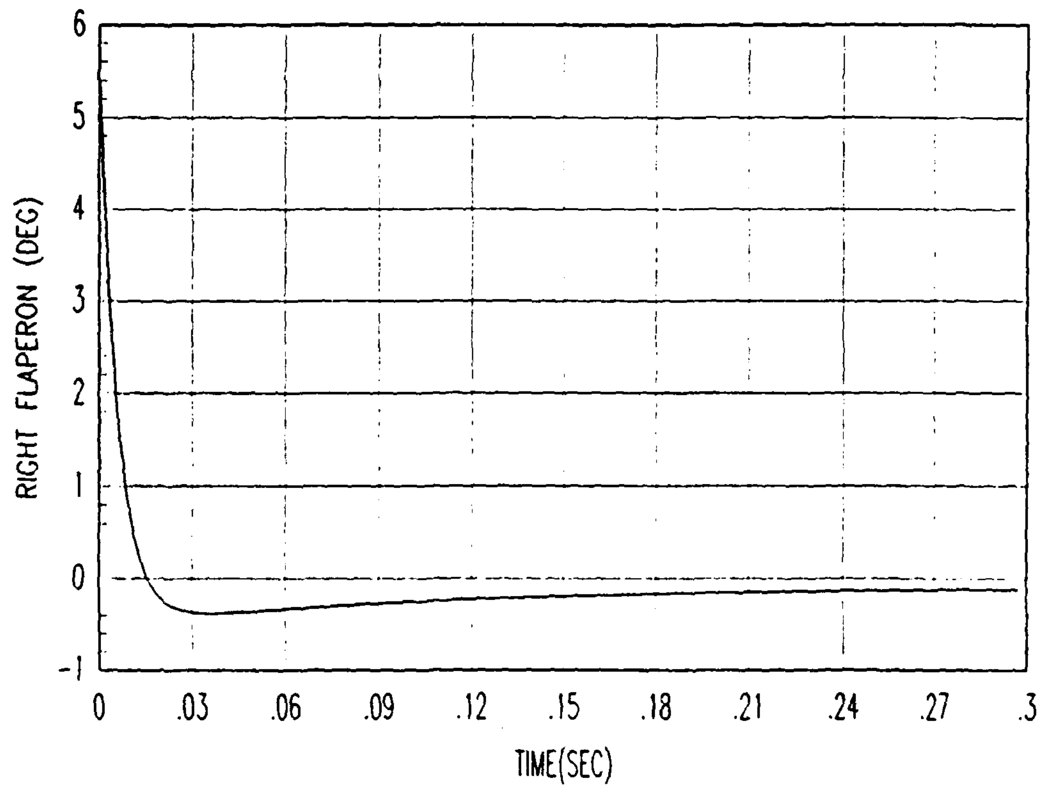

Figure 5.10. Right Flaperon Unit Step Response for Choice #1

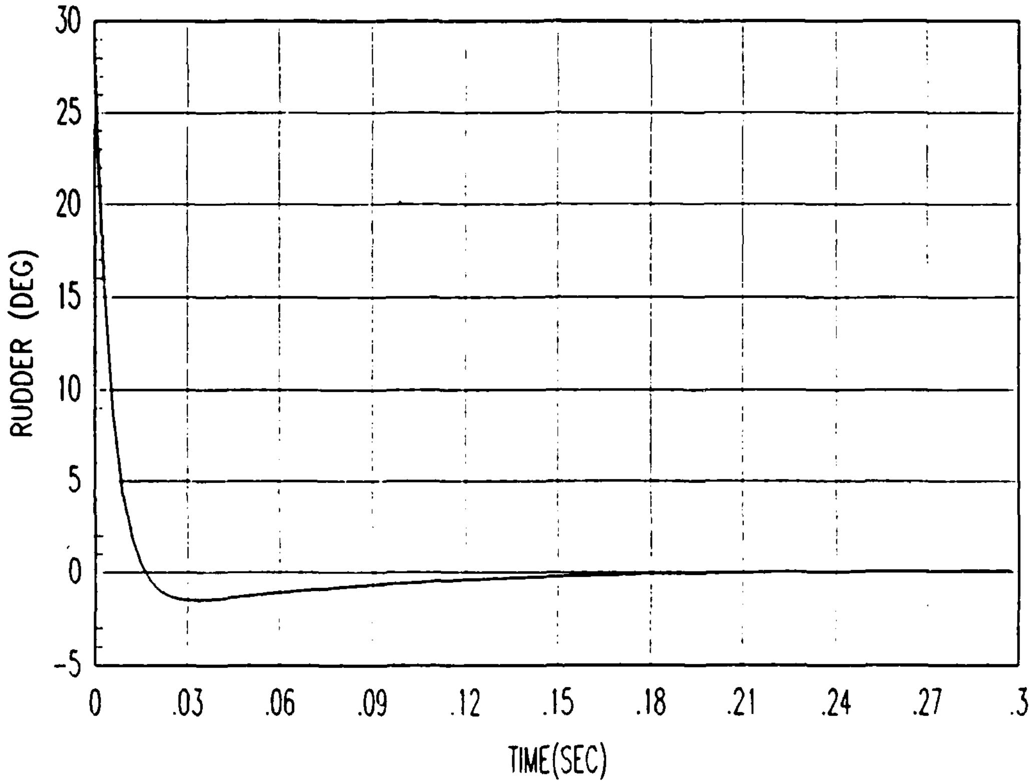

Figure 5.11. Rudder Unit Step Response for Choice #1

The diagonal dominance condition is satisfied for all failures down to 0.01% of available control surface deflections.

Finally, a check of zeros of each det$[P_{e_f}]$ is made. Table 5.6 shows the zeros of det$[P_{e_f}]$ for the failures of Table 5.4. In summary, this choice #1 of $P_e(s)$ is not acceptable in light of the large u_{rf}, u_{lf}, and u_r deflections it commands for unit step inputs of v_1 and v_2.

<u>P_e Choice #2</u>

For $P_e = \begin{bmatrix} \frac{(s+9)}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$, the weighting matrix is

$$\Delta = \frac{\begin{bmatrix} \delta_{n11} & \delta_{n12} \\ \delta_{n21} & \delta_{n22} \\ \delta_{n31} & \delta_{n32} \end{bmatrix}}{(6.0627e-5)s^4 + (1.1213e-2)s^3 + (2.0606e-1)s^2 + s} \tag{5.31}$$

where

$$\delta_{n11} = (3.3591e-4)s^4 + (3.1669e-3)s^3 - (6.6175e-3)s^2 - (7.1199e-2)s \tag{5.32}$$

$$\delta_{n12} = (-1.1966e-4)s^3 - (2.0199e-2)s^2 - (5.4795e-2)s - (3.2570e-4) \tag{5.33}$$

$$\delta_{n21} = -\delta_{n11} \tag{5.34}$$

$$\delta_{n22} = -\delta_{n12} \tag{5.35}$$

$$\delta_{n31} = (1.1650e-3)s^4 + (1.6401e-2)^3 + (2.4856e-2)s^2 + (1.2908e-1)s \tag{5.36}$$

$$\delta_{n32} = (-4.4301e-6)s^3 - (1.3682e-3)^2 - (1.9729e-5)s - (2.0829e-3) \tag{5.37}$$

Figure 5.12 shows desired sideslip β response and Figure 5.9 shows the desired roll rate p response for unit step inputs. Figure 5.13 is similar to Figure 5.10 except that the right flaperon defection has slightly less overshoot for $\Delta(s)$ choice #2 than for $\Delta(s)$ choice #1. Figure 5.14 shows less rudder deflection than Figure 5.11 but such a large rudder deflection of approximately 19° for 1° step inputs in V_1 and V_2 is still unacceptable.

Table 5.6. Zeros of $\det[P_{e_f}]$ for P_e Choice #1

Failure No.	Right-Half Plane Zeros	Zeros at Origin
1		
2	2.9331 $0.0019 + j0.0624$ $0.0019 - j0.0624$	
3	2.7060 $0.4668 + j4.0894$ $0.4668 - j4.0894$ $0.0020 + j0.0624$ $0.0020 - j0.0624$	
4	5.3081 $1.9298 + j2.6871$ $1.9298 - j2.6871$	3
5		
6		
7		
8		
9		
10	4.3312 $0.6170 + j4.6131$ $0.6170 - j4.6131$ $0.1036 + j0.1609$ $0.1036 - j0.1609$	
11		
12		
13		
14		
15		
16		
17		

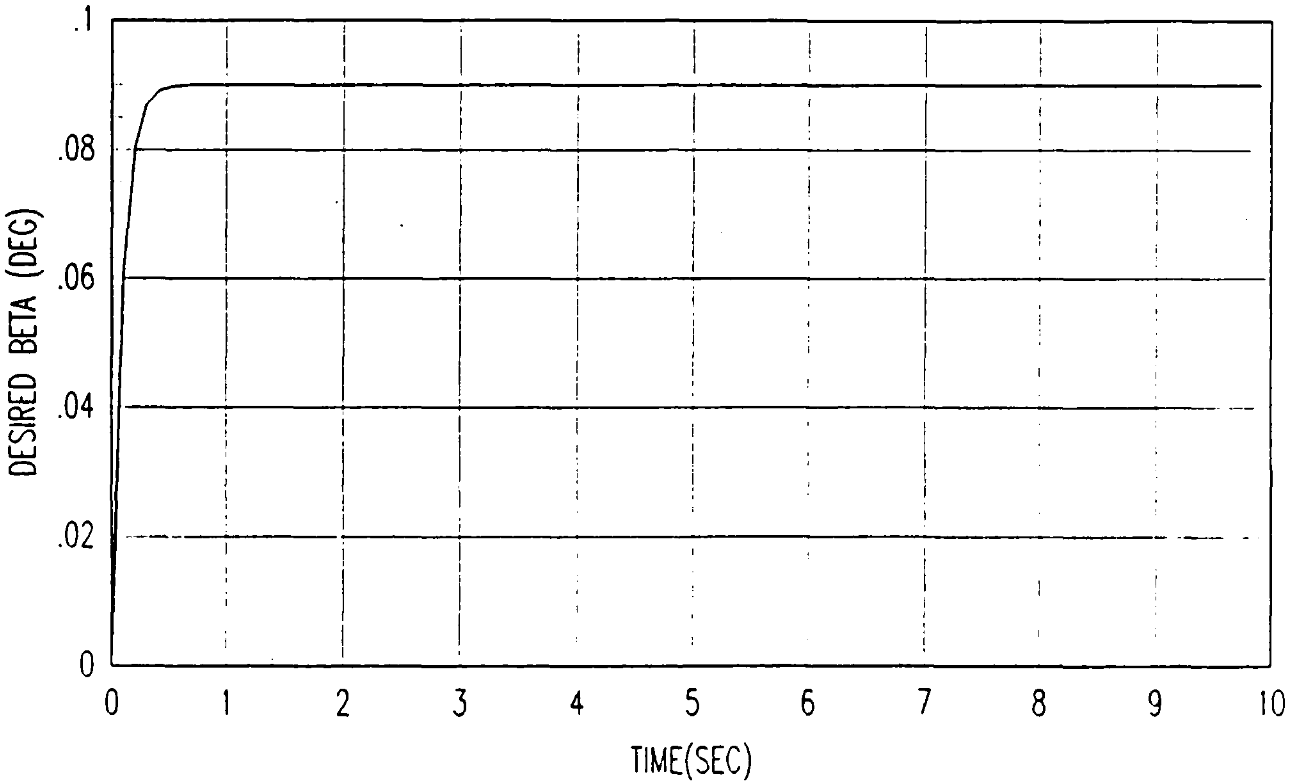

Figure 5.12. Desired Sideslip Unit Step Response for Choice #2

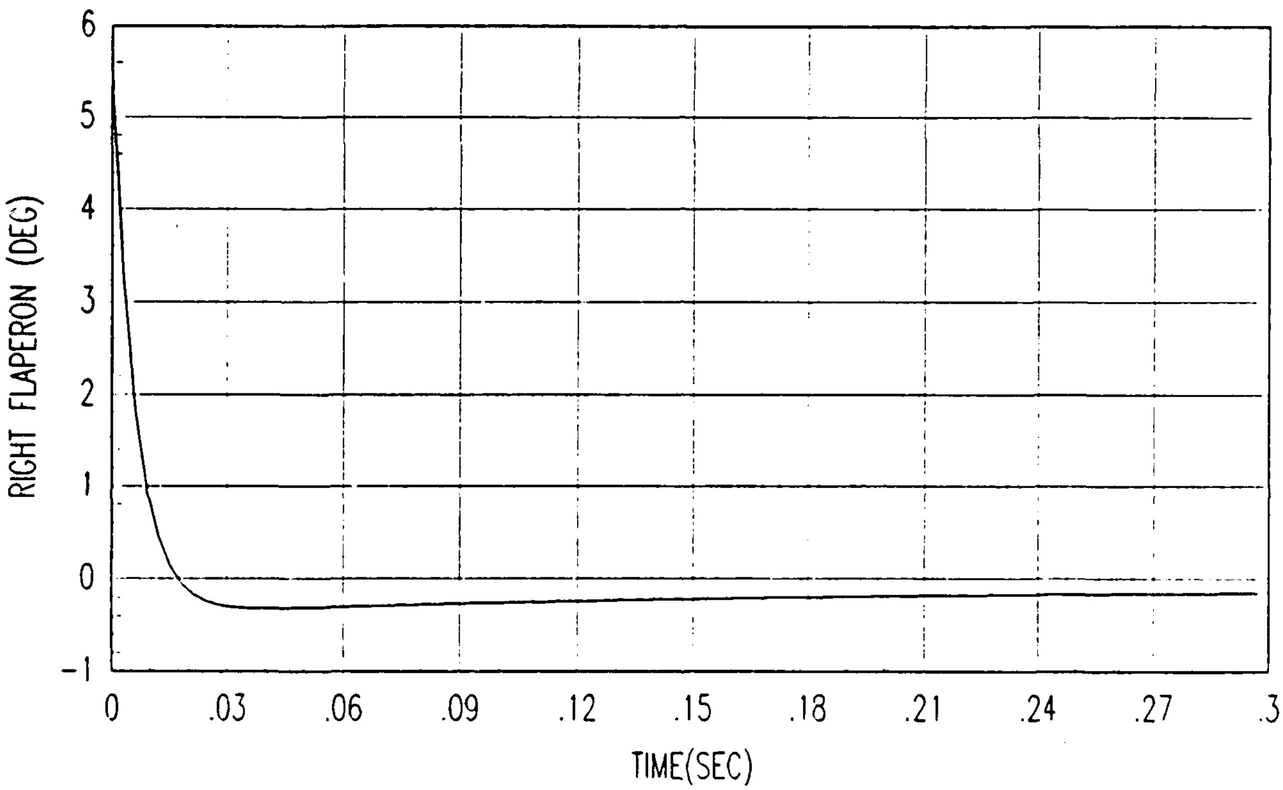

Figure 5.13. Right Flaperon Unit Step Response for Choice #2

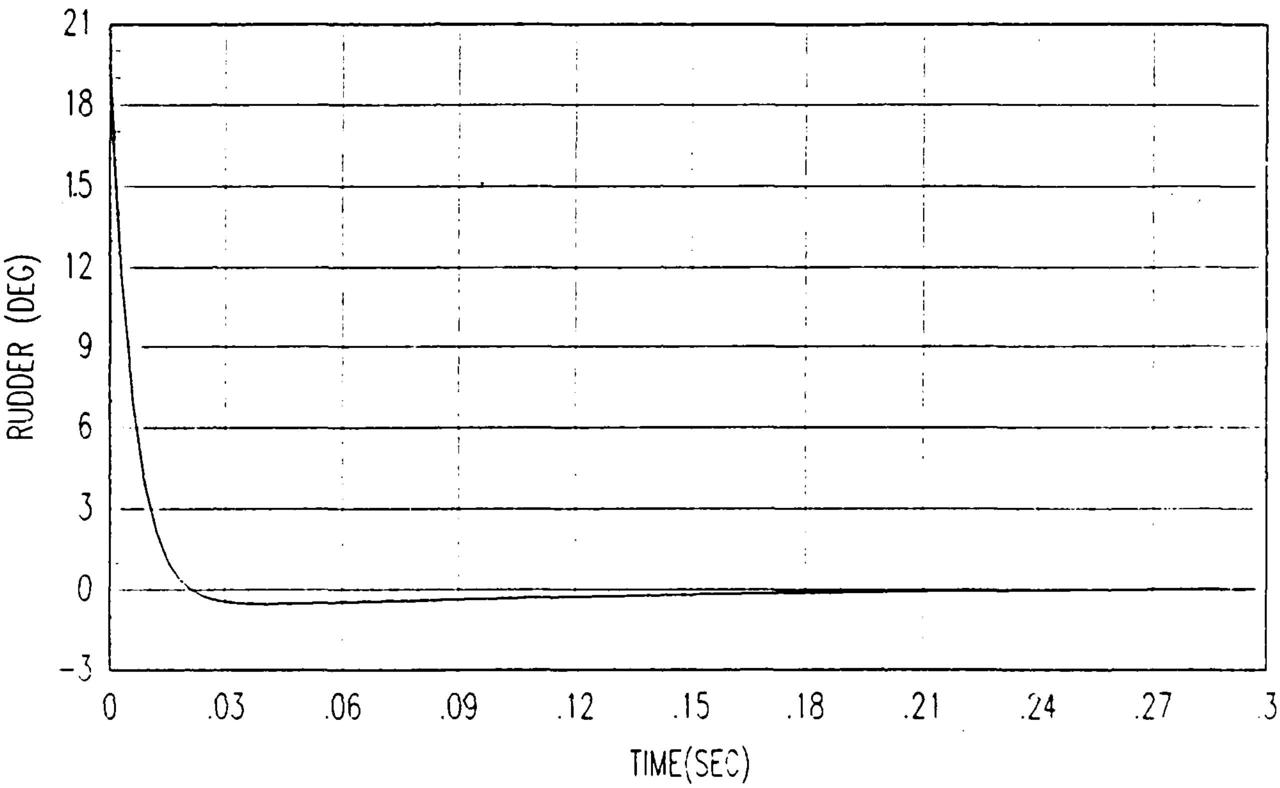

Figure 5.14. Rudder Unit Step Response for Choice #2

Table 5.7. Zeros of $\det[P_{e_f}]$ for P_e Choice #2

Failure No.	Right-Half Plane Zeros	Zeros at Origin
1		
2	9.8535 $0.3633 + j4.4728$ $0.3633 - j4.4728$	5
3	17.4698 1.1118 $0.7381 + j5.1057$ $0.7381 - j5.1057$	3
4	4.000 1.4832	3
5		
6		
7		
8		
9		
10	9.6381 1.2592	
11		
12		
13		
14		
15		
16		
17		

The diagonal dominance condition is satisfied for failures down to 0.01% of available control surface deflections.

Table 5.7 shows the zeros of $\det[P_{e_f}]$ for the 17 failure cases.

In summary, choice #2 of $P_e(s)$ is not acceptable in light of the large deflections of the control surfaces u_{rf}, u_{lf}, and u_r. Also, the "more" minimum-phase $\det[P_e(s)]$ for the healthy $P_e(s)$ actually causes the zeros of $\det[P_{e_f}]$ to be farther in the right-half s plane than choice $P_e(s)$ #1.

For P_e Choice #3.

For $P_e(s) = \begin{bmatrix} \frac{(s+1)}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$, the weighting matrix is

$$\Delta = \frac{\begin{bmatrix} \delta_{n11} & \delta_{n12} \\ \delta_{n21} & \delta_{n22} \\ \delta_{n31} & \delta_{n32} \end{bmatrix}}{(6.0627e-5)s^4 + (1.1213e-2)s^3 + (2.0606e-1)s^2 + s} \tag{5.38}$$

where

$$\delta_{n11} = (3.3591e-4)s^4 + (4.7963e-4)s^3 - (7.7673e-3)s^2 - (7.9110e-3)s \tag{5.39}$$

$$\delta_{n12} = (-1.1966e-4)s^3 - (2.0199e-2)s^2 - (5.4795e-2)s - (3.2570e-4) \tag{5.40}$$

$$\delta_{n21} = -\delta_{n11} \tag{5.41}$$

$$\delta_{n22} = -\delta_{n12} \tag{5.42}$$

$$\delta_{n31} = (1.6496e-3)s^4 + (2.8178e-3)s^3 + (1.5510e-2)s^2 + (1.4342e-2)s \tag{5.43}$$

$$\delta_{n32} = (-4.4301e-6)s^3 - (1.3682e-2)s^2 - (1.9729e-4)s - (2.0829e-3) \tag{5.44}$$

Figures 5.15 and 5.9 show the desired equivalent plant responses to unit step inputs. The right flaperon deflection of Figure 5.16 has a slightly larger overshoot than Figures 5.10 and 5.13. Figure 5.17 shows an excessively large rudder deflection similar to Figure 5.11.

The diagonal dominance condition is satisfied down to failures of 0.01% of available surface deflections.

Table 5.8 shows the zeros of the determinant of the failed equivalent plant.

In summary, choice #3 for $P_e(s)$ is unacceptable in light of the excessive control deflections commanded by $\Delta(s)$. Also, the "less" minimum-phase healthy $\det[P_e(s)]$ does not improve the location of the r.h.p zeros of $\det[P_{e_f}(s)]$ for the failure cases involving complete surface failures.

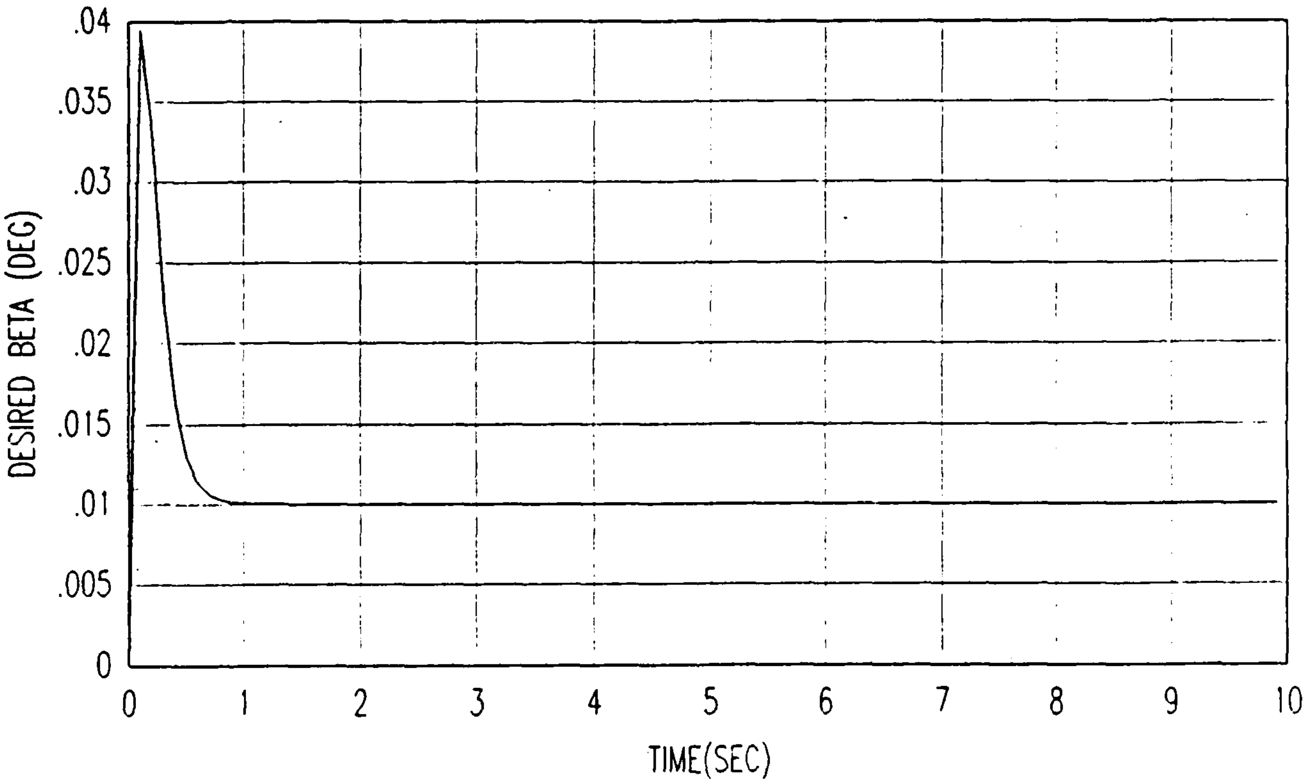

Figure 5.15. Desired Sideslip Unit Step Response for Choice #3

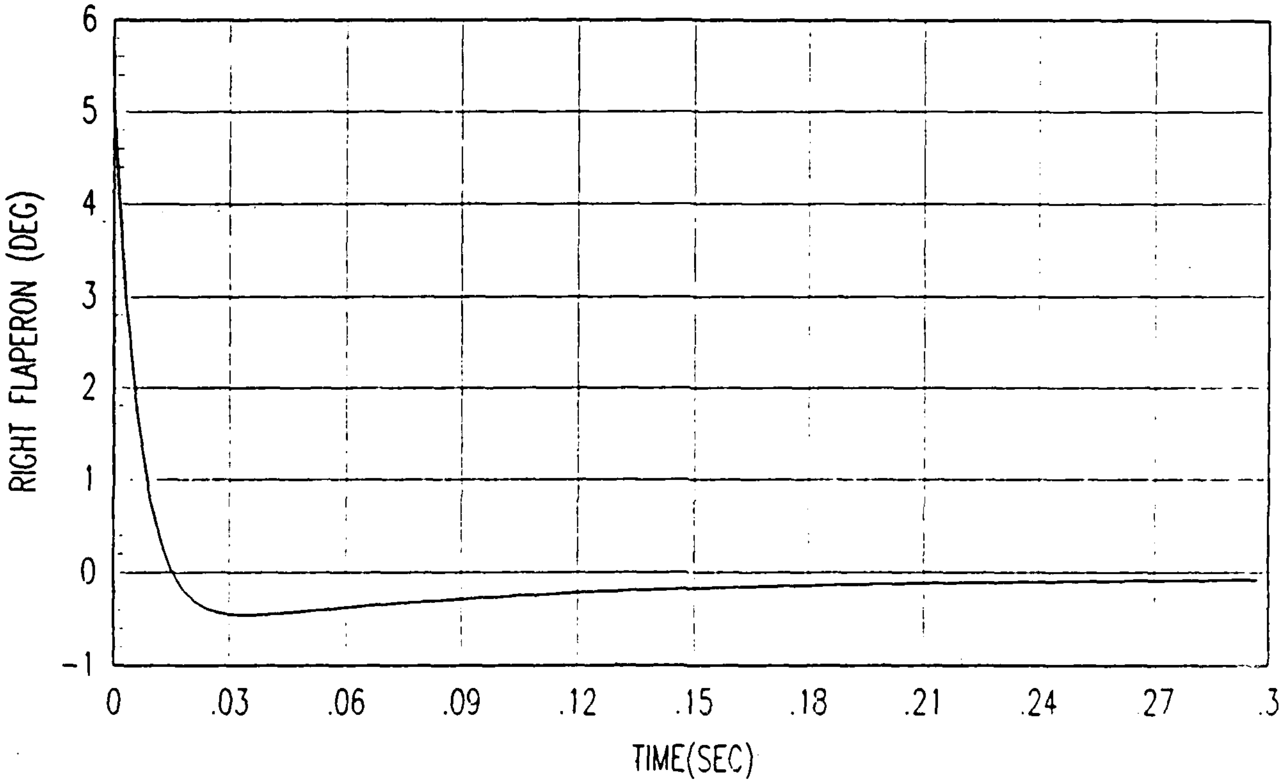

Figure 5.16. Right Flaperon Unit Step Response for Choice #3

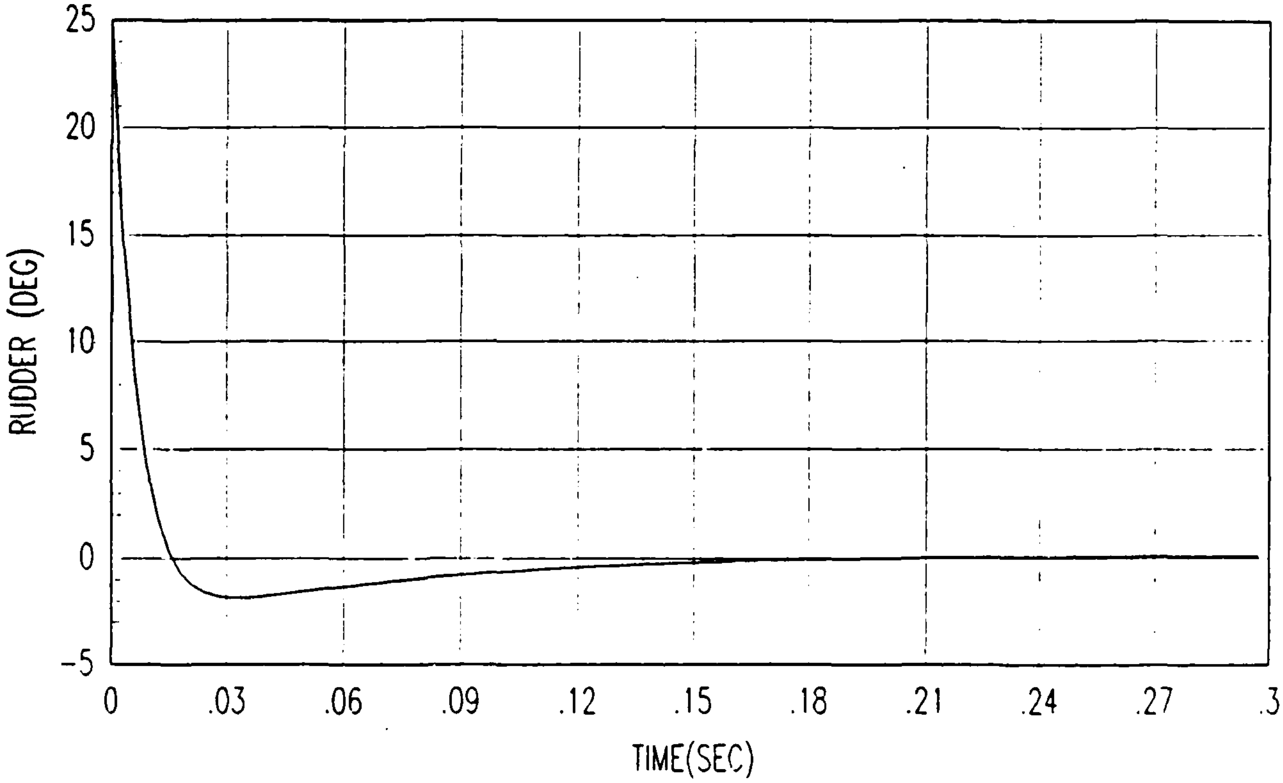

Figure 5.17. Rudder Unit Step Response for Choice #3

Table 5.8. Zeros of $\det[P_{e_f}]$ for P_e Choice #3

Failure No.	Right-Half Plane Zeros	Zeros at Origin
1		
2	8.0687 $0.3956 + j1.2675$ $0.3956 - j1.22675$	2
3	6.6973 $0.3657 + j1.5310$ $0.3657 - j1.5310$	2
4	5.2977 $0.9213 + j3.1922$ $0.9213 - j3.1922$ $0.3507 + j0.9407$ $0.3507 - j0.9407$	
5		
6		
7		
8		
9		
10	30.9174 1.2606 0.4766 $4.4046 + j28.9746$ $4.4046 - j28.9746$ $0.1071 + j0.4238$ $0.1071 - j0.4238$	
11		
12		
13		
14		
15		
16		
17		

For P_e Choice #4

For $P_e = \begin{bmatrix} \frac{(s+50)}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$, the weighting matrix is

$$\Delta = \frac{\begin{bmatrix} \delta_{n11} & \delta_{n12} \\ \delta_{n21} & \delta_{n22} \\ \delta_{n31} & \delta_{n32} \end{bmatrix}}{(6.0627e-5)s^4 + (1.1213e-2)s^3 + (2.0606e-1)s^2 + s} \tag{5.45}$$

where

$$\delta_{n11} = (3.3591e-4)s^4 + (1.6939e-2)s^3 - (7.24910e-4)s^2 - (3.9555e-1)s \tag{5.46}$$

$$\delta_{n12} = (-1.1966e-4)s^3 - (2.0199e-2)s^2 - (5.4795e-2)s - (3.2570e-4) \tag{5.47}$$

$$\delta_{n21} = -\delta_{n11} \tag{5.48}$$

$$\delta_{n22} = -\delta_{n12} \tag{5.49}$$

$$\delta_{n31} = (1.6496e-3)s^4 + (8.3648e-2)s^3 + (7.2751e-2)s^2 + (7.1710e-1)s \tag{5.50}$$

$$\delta_{n32} = (-4.4301e-6)s^3 - (1.3682e-3)s^2 - (1.9729e-4)s - (2.0829e-3) \tag{5.51}$$

Figures 5.18 and 5.9 show the desired equivalent plant responses for unit step inputs in v_1 and v_2. The right flaperon deflection of Figure 5.19 does not reach 0° as quickly as Figures 5.10, 5.13, and 5.16. The rudder deflection of Figure 5.20 is similar to that of Figure 5.11 except that u_r of Figure 5.20 does not reach 0° as quickly.

The diagonal dominance condition is satisfied for all failures other than ones involving 100% surface failure.

Table 5.9 shows the zeros of det$[P_{e_f}]$ for the failures considered.

In summary, $P_e(s)$ choice #4 is unacceptable in light of the large surface deflections commanded by $\Delta(s)$. Also, the presence of $(s+50)$ in the numerator of $p_{e_{11}}(s)$ does not appreciably improve the location of the r.h.p. zeros of det$[P_{e_f}(s)]$ for the failures considered.

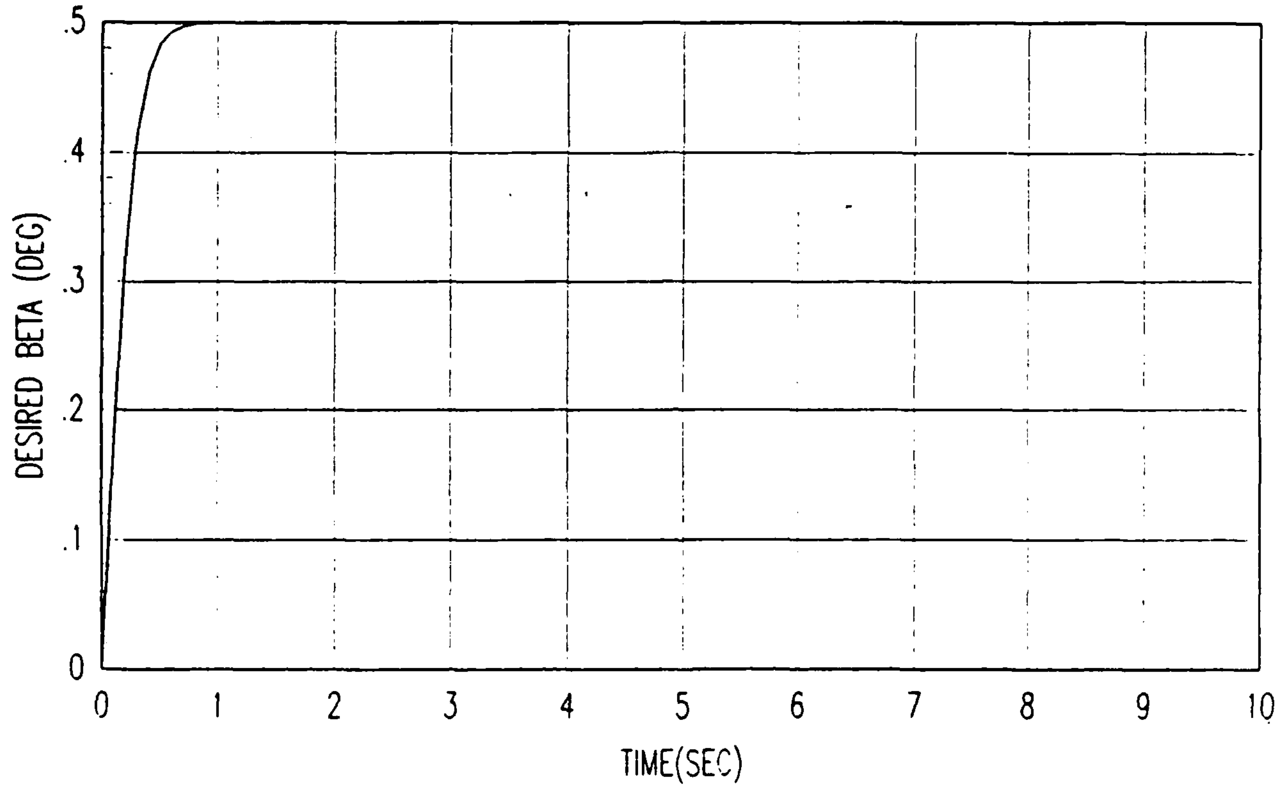

Figure 5.18. Desired Sideslip Unit Step Response for Choice #4

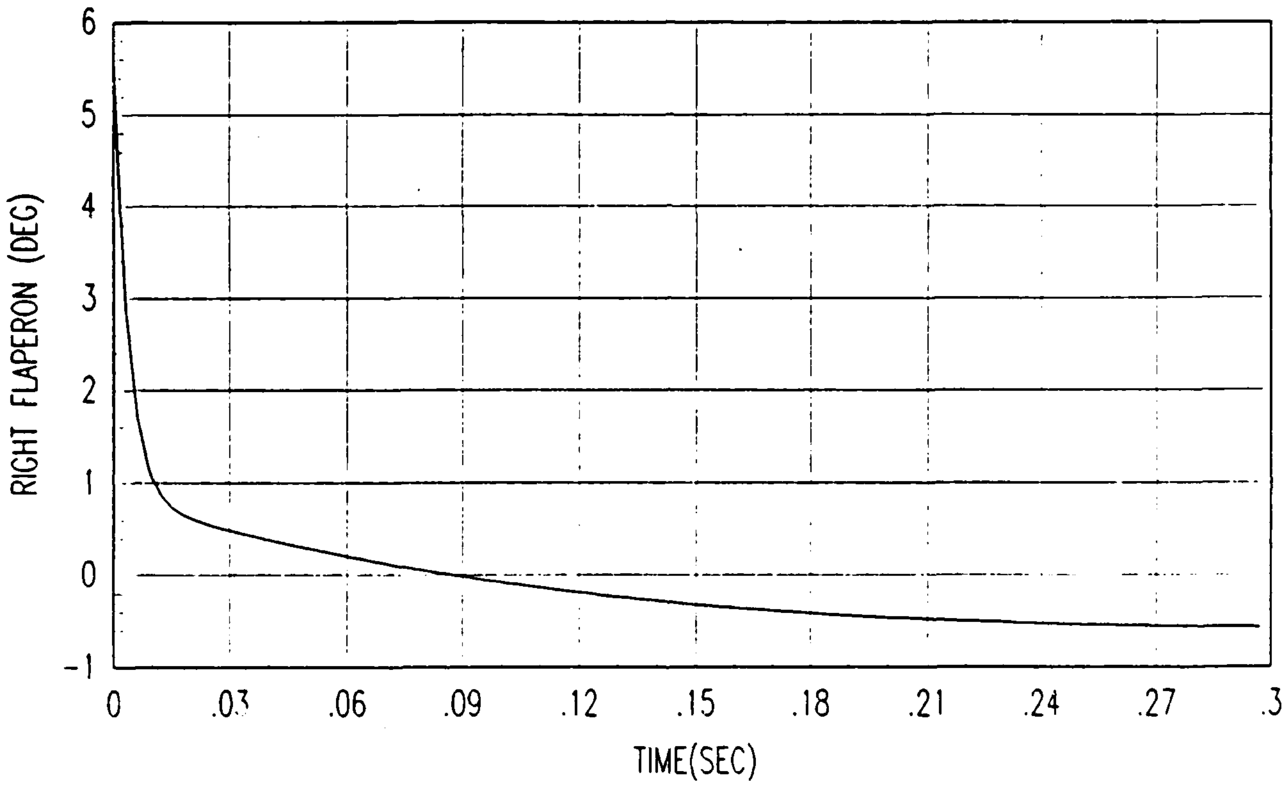

Figure 5.19. Right Flaperon Unit Step Response for Choice #4

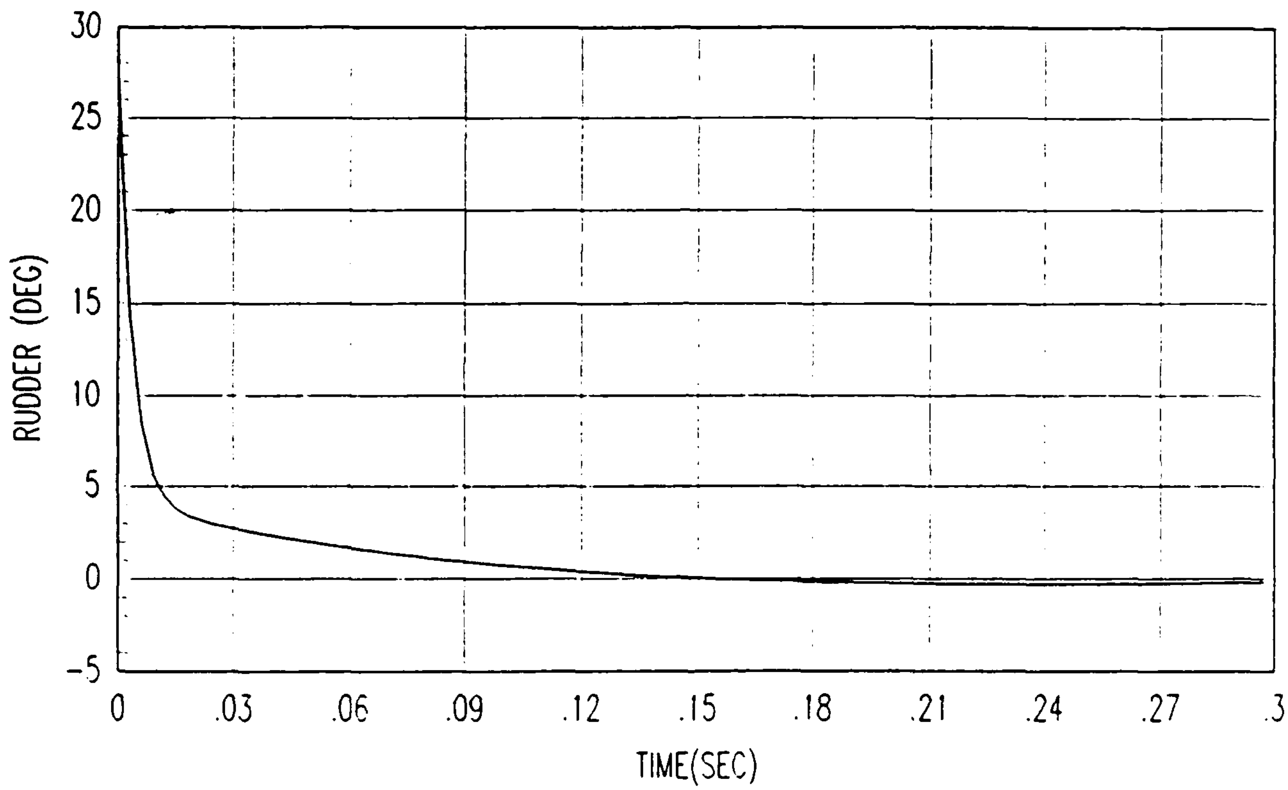

Figure 5.20. Rudder Unit Step Response for Choice #4

Table 5.9. Zeros of $\det[P_{e_f}]$ for P_e Choice #4

Failure No.	Right-Half Plane Zeros	Zeros at Origin
1		
2	$2.1417 + j2.8329$ $2.1417 - j2.8329$	1
3	$2.6941 + j1.6598$ $2.6941 - j1.6598$ $0.8003 + j1.8474$ $0.8003 - j1.8474$	1
4	207.1 4.9349 $0.0034 + j2.2581$ $0.0034 - j2.2581$	4
5		
6		
7		
8		
9		
10	11.9695 $1.2710 + j1.4797$ $1.2710 - j1.4797$	
11		
12		
13		
14		
15		
16		
17		

For P_e Choice #5

For $P_e = \begin{bmatrix} \frac{0.01}{(s+10)^2} & 0 \\ 0 & \frac{100}{(s+10)^2} \end{bmatrix}$, the weighting matrix is

$$\Delta = \frac{\begin{bmatrix} \delta_{n11} & \delta_{n12} \\ \delta_{n21} & \delta_{n22} \\ \delta_{n31} & \delta_{n32} \end{bmatrix}}{(6.0627e-5)s^4 + (1.1213e-2)s^3 + (2.0606e-1)s^2 + s} \quad (5.52)$$

where

$$\delta_{n11} = (3.3591e-6)s^3 + (1.4372e-6)s^2 - (7.9110e-5)s \quad (5.53)$$

$$\delta_{n12} = (-1.1966e-4)s^3 - (2.0199e-2)s^2 - (5.4795e-2)s - (3.2570e-4) \quad (5.54)$$

$$\delta_{n21} = -\delta_{n11} \quad (5.55)$$

$$\delta_{n22} = -\delta_{n12} \quad (5.56)$$

$$\delta_{n31} = (1.6496e-5)s^3 + (1.1682e-5)s^2 + (1.4342e-4)s \quad (5.57)$$

$$\delta_{n32} = (-4.4301e-6)s^3 - (1.3682e-3)s^2 - (1.9729e-4)s - (2.0829) \quad (5.58)$$

Figures 5.21 and 5.9 show the desired equivalent plant responses to unit step inputs in v_1 and v_2. Figures 5.22 and 5.23 show the right flaperon deflection for unit step inputs in v_1 and v_2. Note the reasonable magnitude of u_{rf} for the 1° step inputs. Figures 5.24 and 5.25 show the rudder deflection for unit step inputs in v_1 and v_2. Note the reasonable magnitude of u_r for the 1° step inputs. In terms of reasonable response for reasonable inputs, choice #5 for $P_e(s)$ is acceptable.

All failed equivalent plants are non-singular for all failures not involving 0% surface failures. Also, the diagonal dominance condition is satisfied for all failures not involving 0% surface failures.

Table 5.10 shows the zeros of $\det[P_{e_f}(s)]$. Note that $\det[P_e(s)]$ for the healthy $P_e(s)$ has no zeros whatsoever. Yet, the failures involving 100% surface failures continue to have r.h.p zeros.

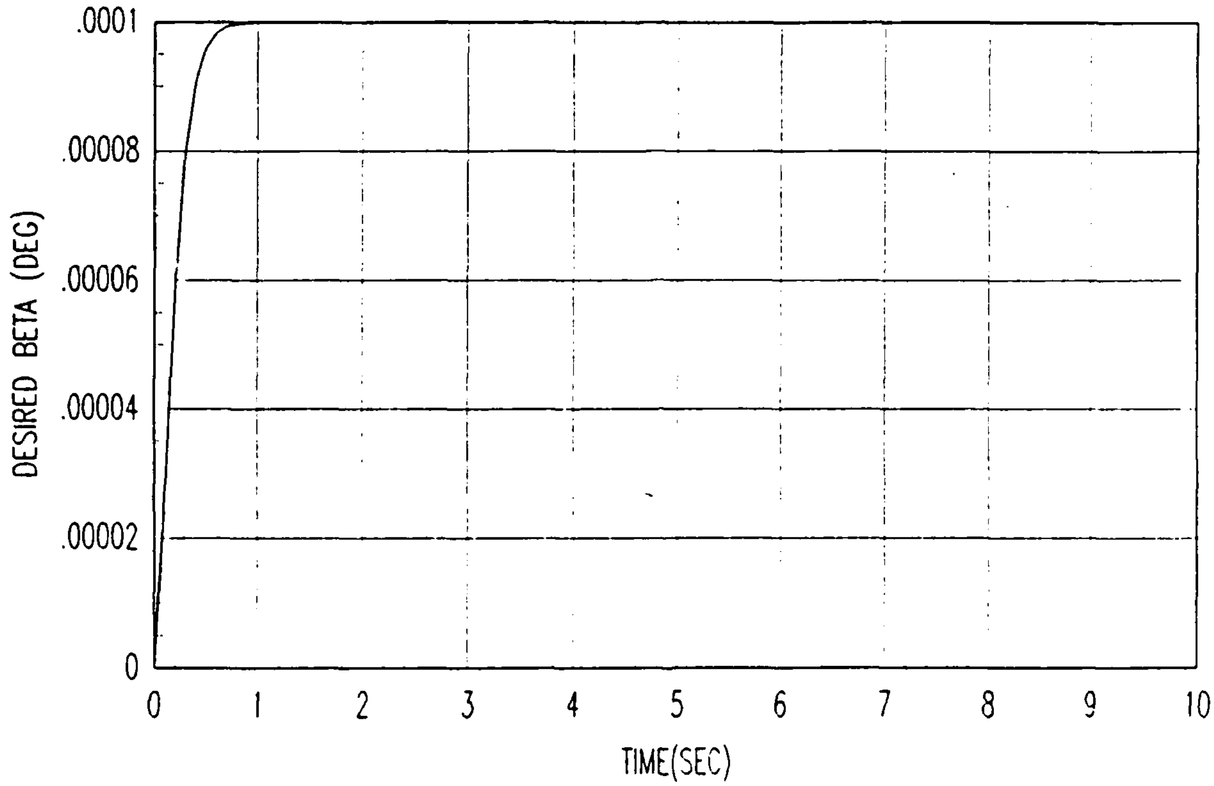

Figure 5.21. Desired Sideslip Unit Step Response for Choice #5

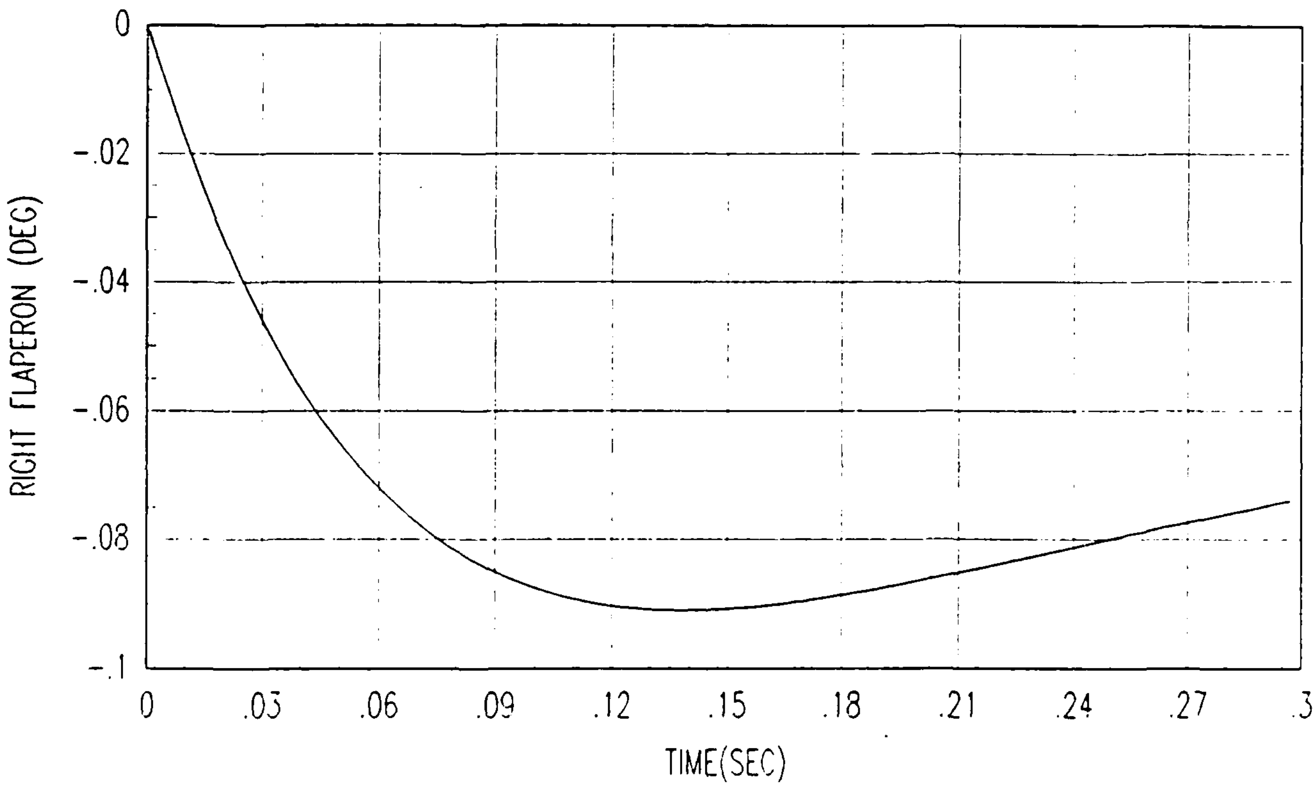

Figure 5.22. Right Flaperon Unit Step Response for Choice #5

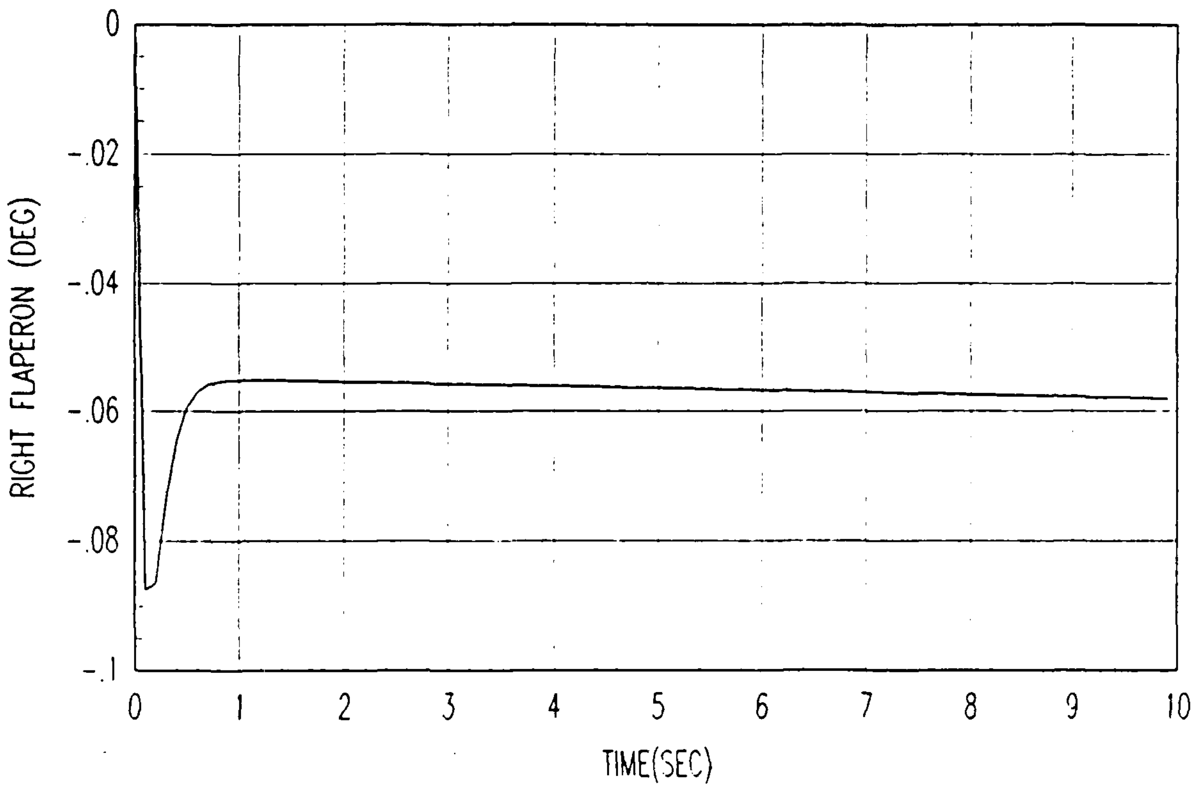

Figure 5.23. Right Flaperon Unit Step Response for Choice #5

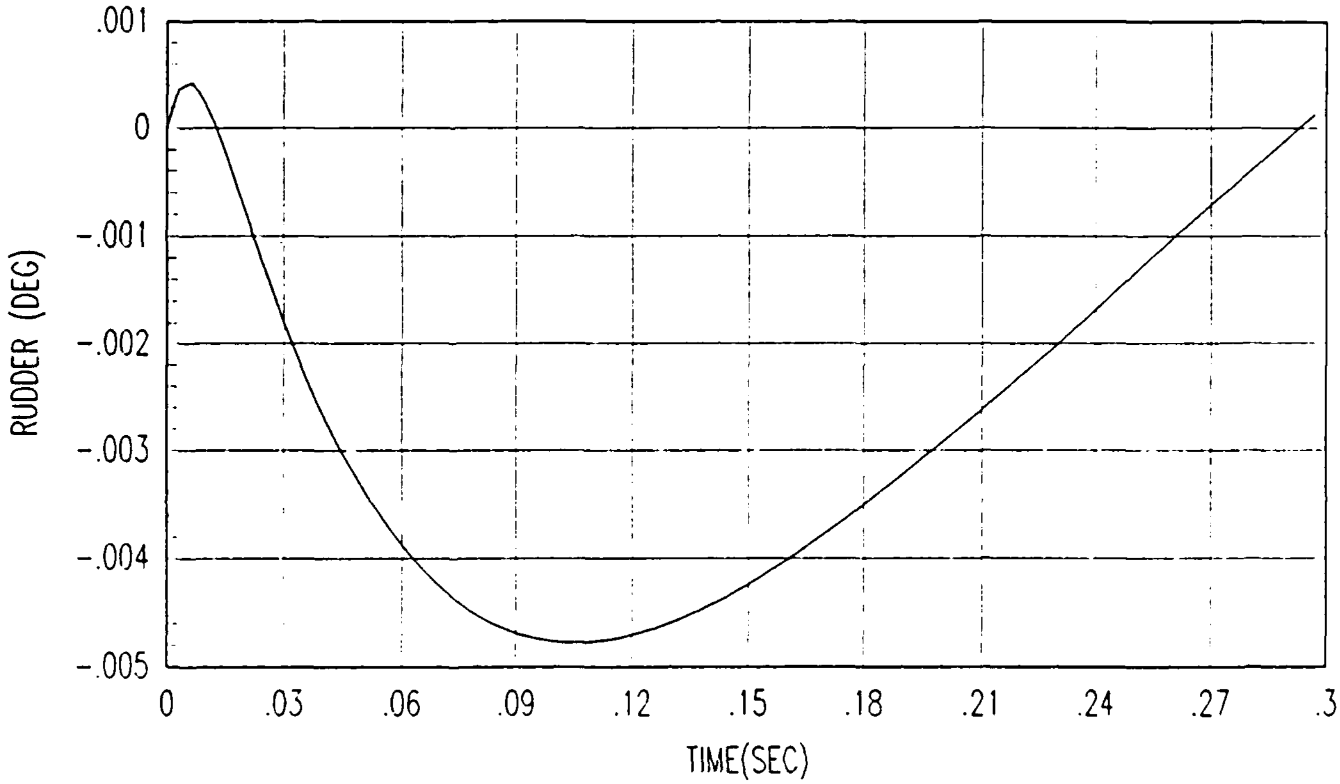

Figure 5.24. Rudder Unit Step Response for Choice #5

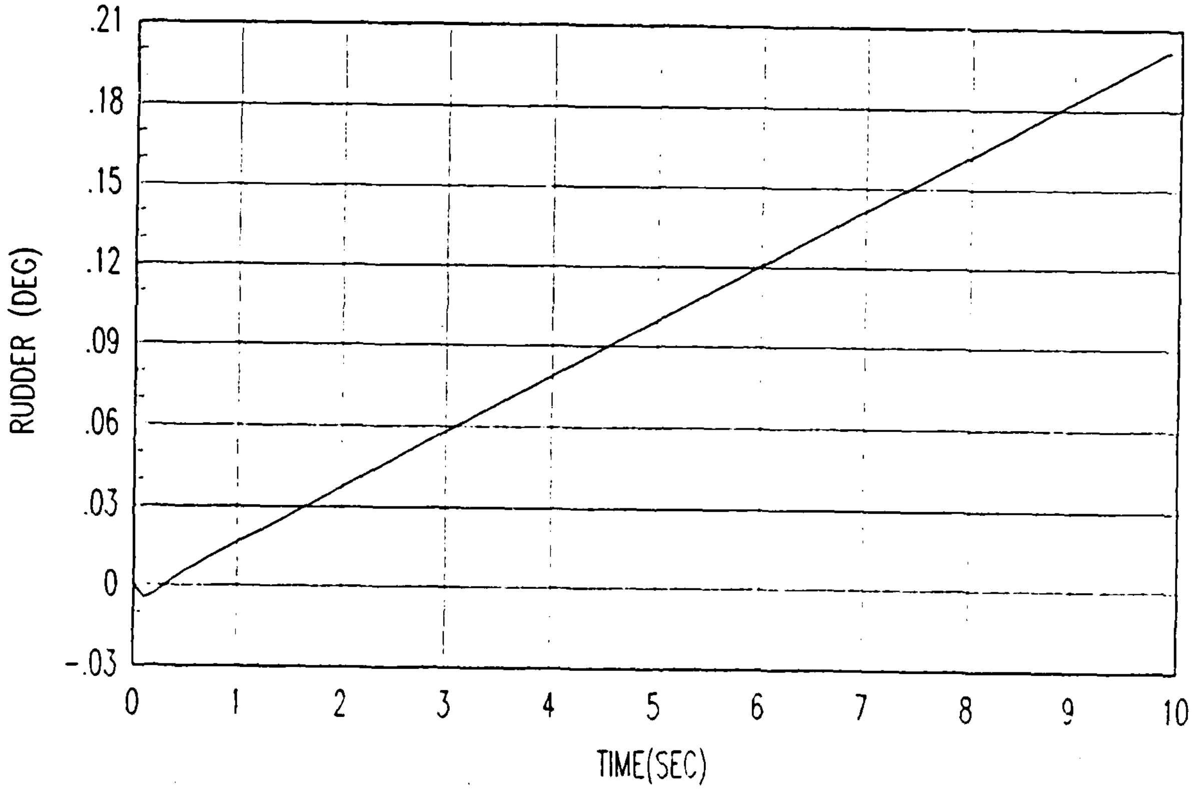

Figure 5.25. Rudder Unit Step Response for Choice #5

Table 5.10. Zeros of $\det[P_{e_f}(s)]$ for $P_e(s)$ Choice #5

Failure No.	Right-Half Plane Zeros	Zeros at Origin
1		
2	2.6208 0.4624	
3	$0.5214 + j4.2817$ $0.5214 - j4.2817$ $1.3572 + j0.8881$ $1.3572 - j0.8881$	
4	123.8 6.8767 1.8558 $0.2456 + j2.0183$ $0.2456 - j2.0183$	1
5		
6		
7		
8		
9		
10	$18.5606 + j13.4305$ $18.5606 - j13.4305$ $(7.6e-6) + j2.0$ $(7.6e-6) - j2.0$	
11		
12		
13		
14		
15		
16		
17		

Based on the above, the weighting matrix of Eq. (5.52) produces acceptable results for failures up to 99.99% of available control surface deflections. For comparison with the healthy equivalent plant unit step response Figures 5.26 and 5.27, the unit step responses of failed equivalent plants #12, #13, and #14 are shown in Figures 5.28 thru 5.33. Recall the failure cases are listed in Table 5.4.

5.6.2 <u>*Summary*</u> All the choices for $P_e(s)$ provide some open-loop performance improvement over the uncompensated plant $P(s)$ of the lateral-directional model of the AFTI/F-16 at Mach 0.9 at 20,000 feet. For all healthy plants, the weighting matrix generated by the Method of Specified Outputs provides the necessary and desired structure to the equivalent plant for the QFT design process to proceed. The desired characteristic of minimum phase $\det[P_{e_f}(s)]$ for all plant failures can not be met with the choices of $P_e(s)$ considered. However, failures up to 0.01% of available control surface deflection are tolerable when using the weighting matrix $\Delta(s)$ generated from the equivalent plant matrix $P_e(s)$ of choice #5 of Table 5.5. Thus, for failures up to 0.01% of available control authority, the Method of Specified Outputs generates a frequency sensitive weighting matrix $\Delta(s)$ that transformed the original plant matrix $P(s)$ into and equivalent plant matrix $P_e(s)$ that possesses the necessary and desired characteristics for the QFT design to proceed.

Note that failures 2,3,4, and 10 of Table 5.4 cause the $\det[P_{e_f}]$'s to be non-minimum phase for all 5 choices of $P_e(s)$. Also, these failures cause $P_{e_f}(s)$ to be almost singular. Failures 2,3 and 10 involve complete loss of the rudder. Failure 4 involves complete loss of both flaperons. On the other hand failure 1, which involves the loss of only one flaperon while the other independent flaperon remains healthy, results in all $\det[P_{e_f}]$'s having no r.h.p. zeros and is thus minimum-phase for all 5 choices of $P_e(s)$. Further exploration is required to find an equivalent plant that will make all $\det[P_{e_f}]$'s minimum-phase.

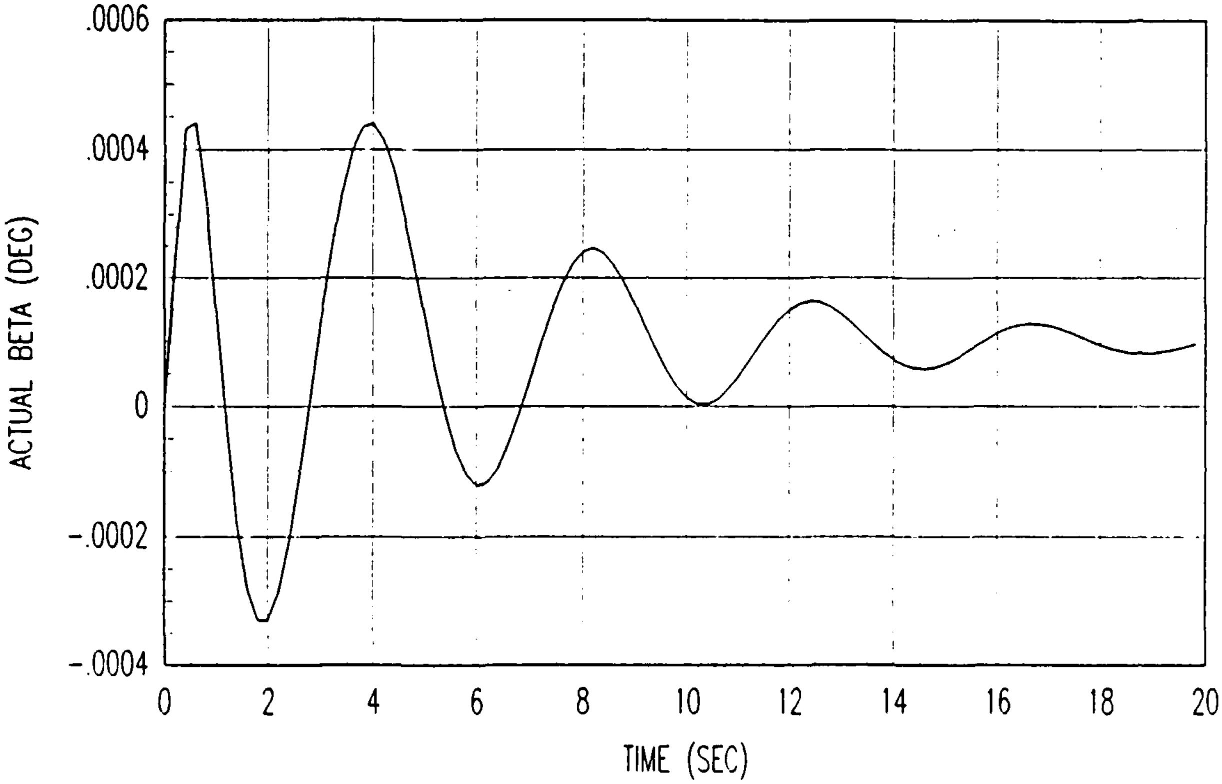

Figure 5.26. Actual Sideslip Unit Step Response for Choice #5

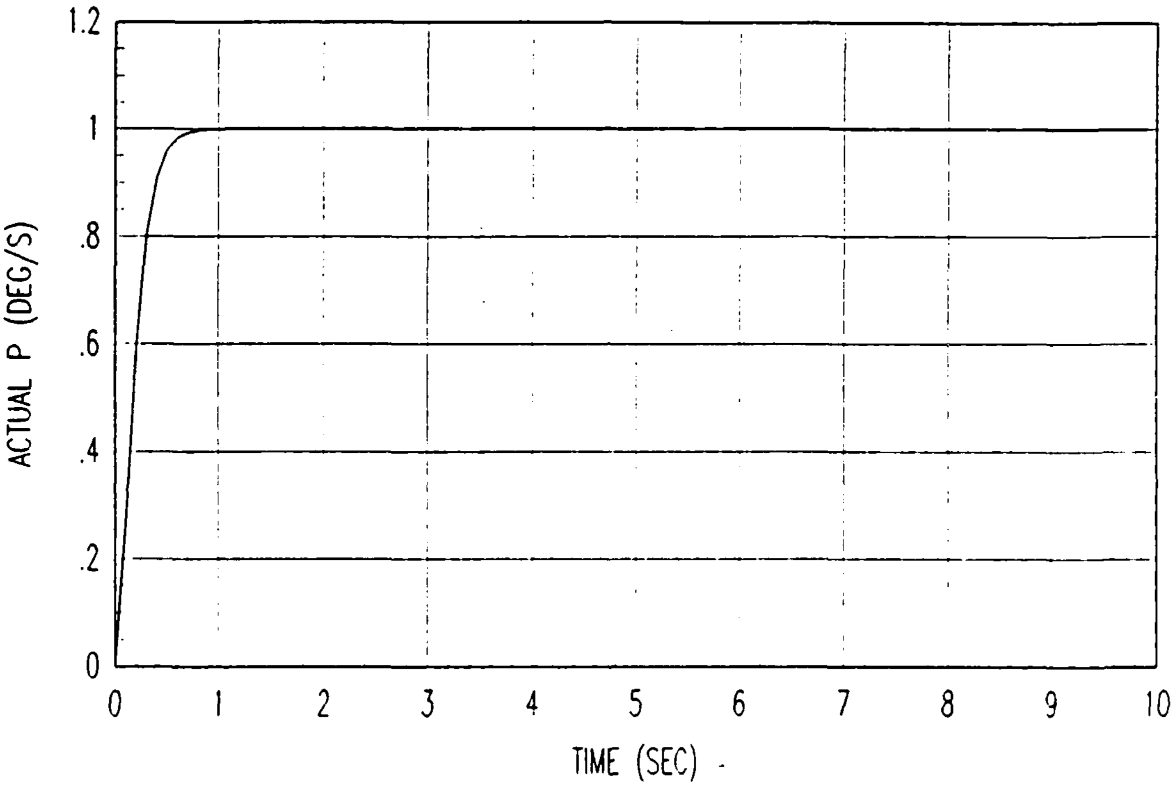

Figure 5.27. Actual Roll Rate Unit Step Response for Choice #5

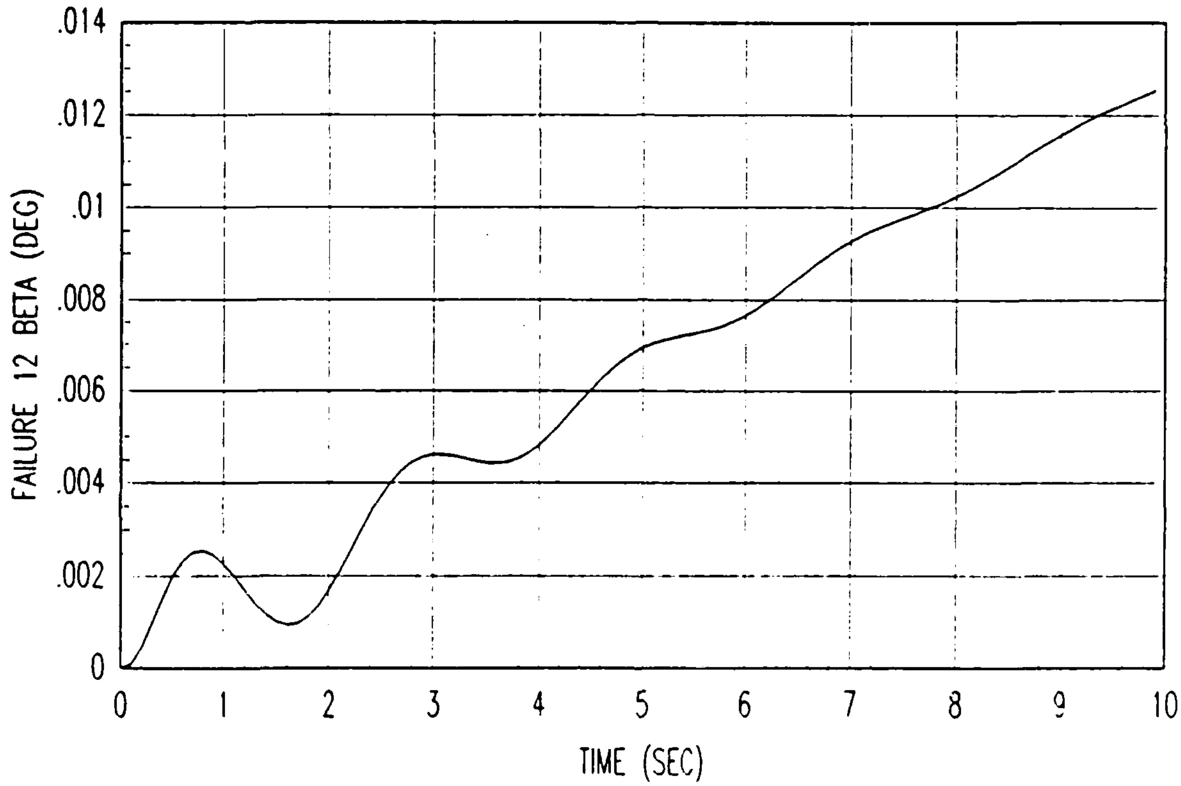

Figure 5.28. Sideslip Unit Step Response for P_{e_f} Failure #12

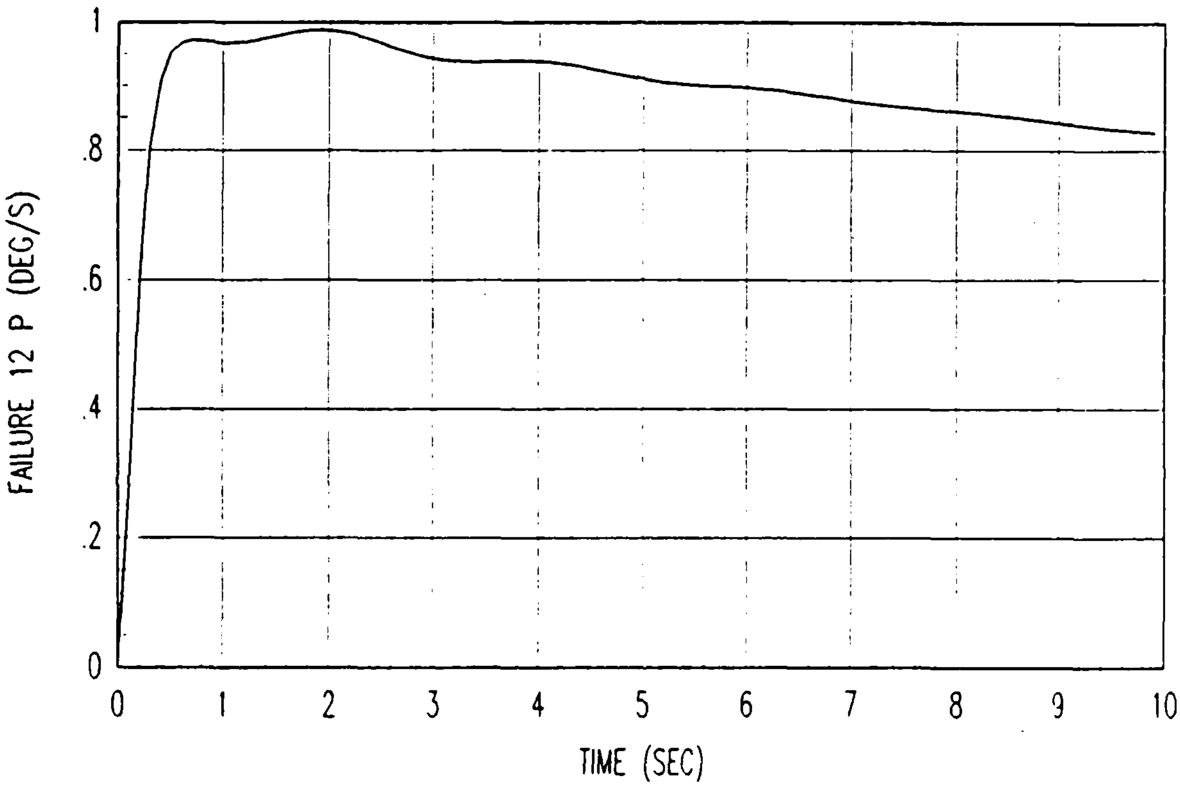

Figure 5.29. Roll Rate Unit Step Response for P_{e_f} Failure #12

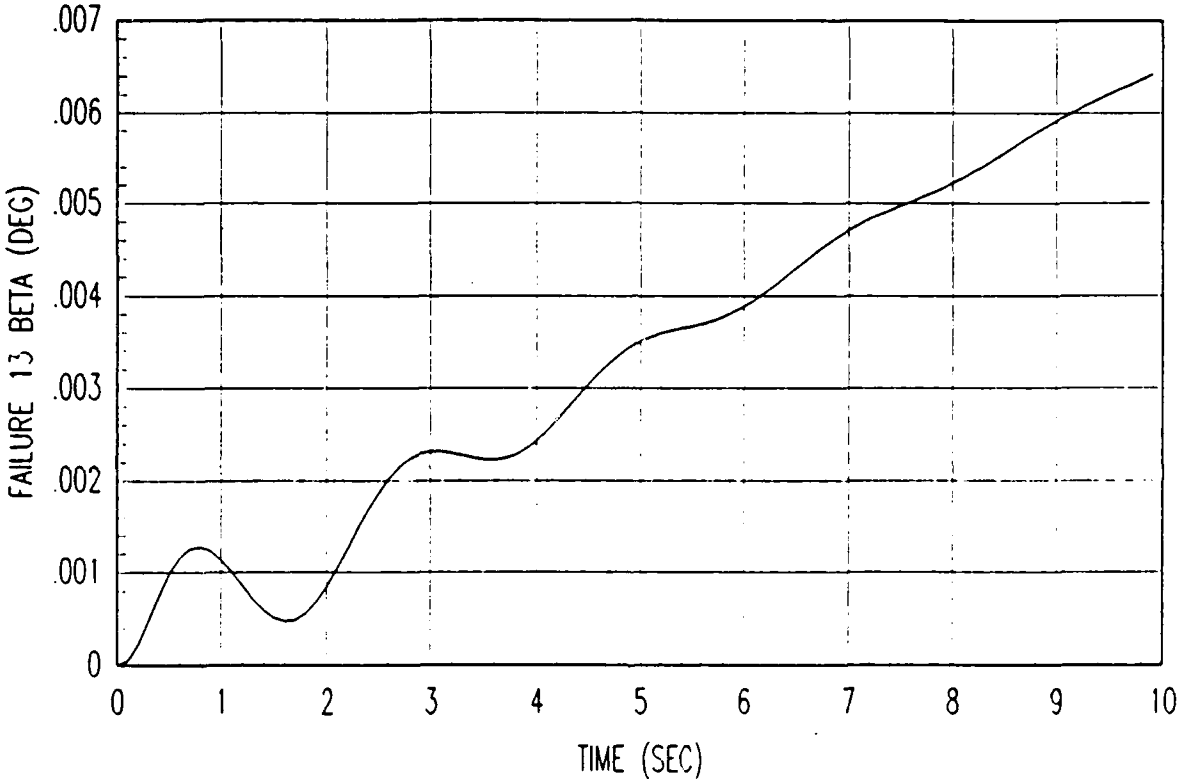

Figure 5.30. Sideslip Unit Step Response for P_{e_f} Failure #13

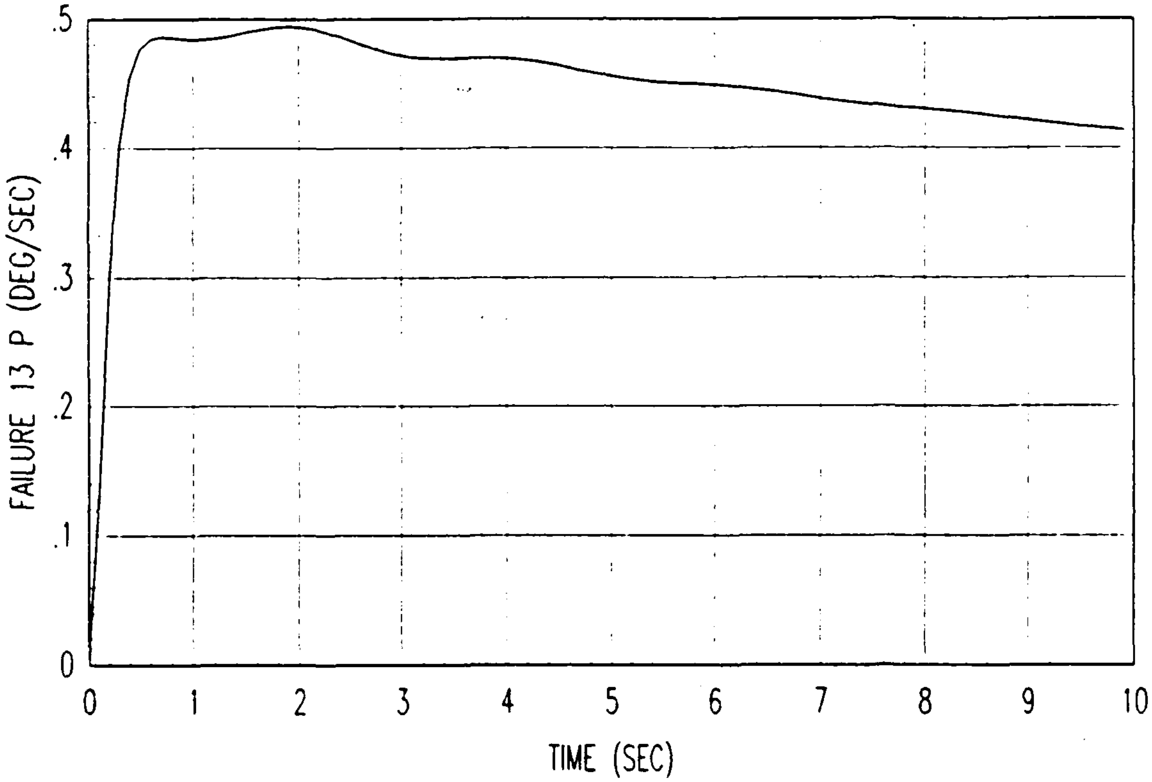

Figure 5.31. Roll Rate Unit Step Response for P_{e_f} Failure #13

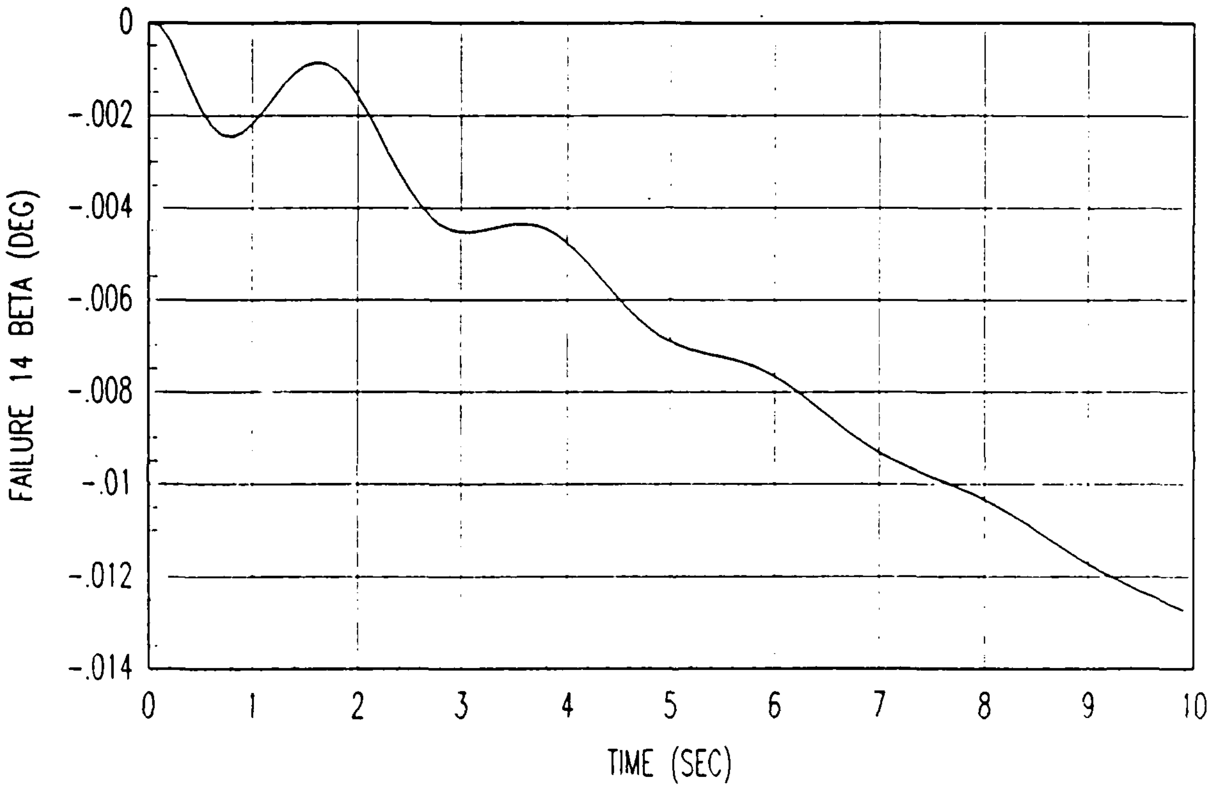

Figure 5.32. Sideslip Unit Step Response for P_{e_f} Failure #14

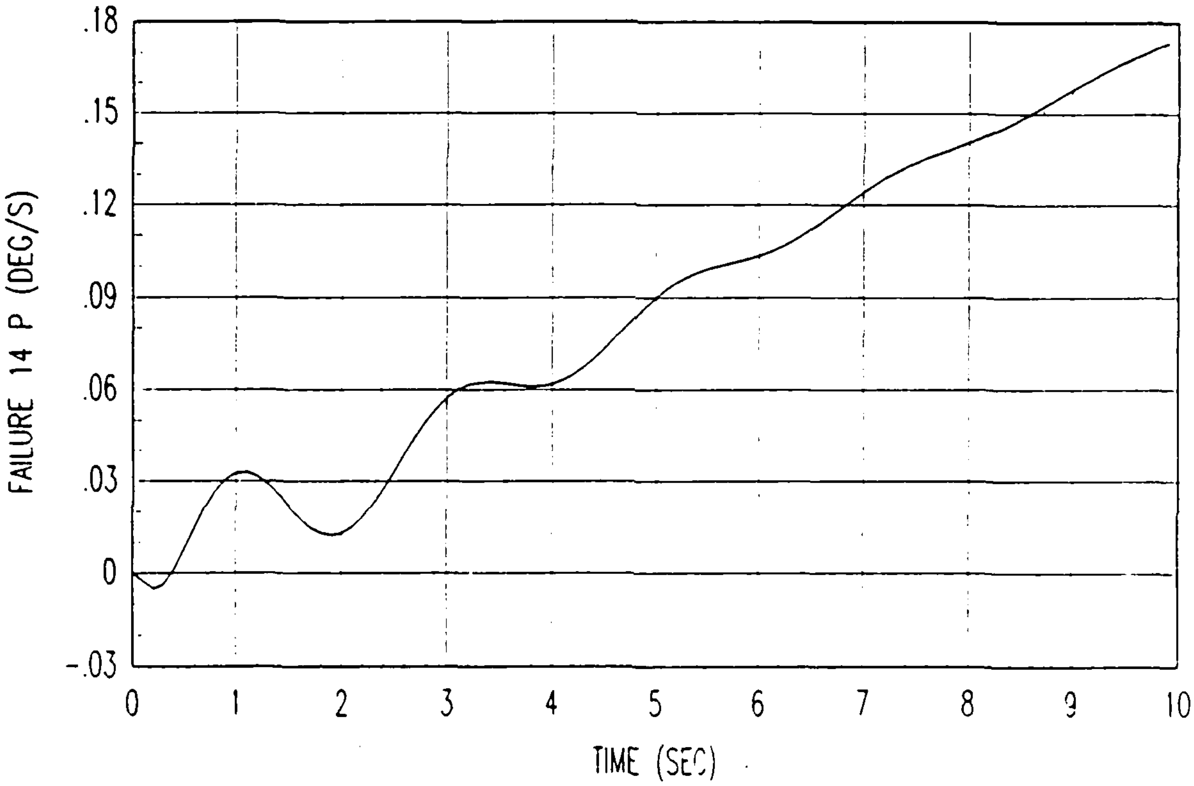

Figure 5.33. Roll Rate Unit Step Response for P_{e_f} Failure #14

VI. Conclusions and Recommendations

6.1 Conclusions

The Method of Specified Outputs is successful in generating a frequency sensitive weighting matrix $\Delta(s)$ for use in the MIMO design process. However, insuring that $\Delta(s)$ commands reasonable control variables, insuring that the diagonal dominance condition is met, and insuring that all $\det[P_{e_f}(s)]$ are minimum phase still requires a trial-and-error approach. The advantage of this trial-and-error approach, when compared to earlier weighting matrix selection techniques, is that choices for the equivalent plant constitutes the trial-and-error and not the choices of the weighting matrix itself. In cases where the number of system outputs is less than or equal to the number of control inputs, choosing a desired equivalent plant with the necessary and desired QFT characteristics does guarantee the actual, healthy equivalent plant will possess the necessary and desired QFT properties.

For the AFTI/F-16 example considered, a frequency sensitive weighting matrix $\Delta(s)$ is found that transforms the original plant matrix $P(s)$ into an equivalent plant matrix $P_e(s)$ that satisfies all QFT necessary and desired characteristics for all failures up to a 99.99% loss in control surface deflection capability. Complete restriction of travel of either the rudder or both flaperons results in failed equivalent plants that do not satisfy the QFT requirements. Once the transfer function of the weighting matrix has been determined numerically, use of a symbolic mathematical tool like MACSYMA allows the QFT designer to quickly compare the affects of different weighting matrices on the original plant matrix.

6.2 Recommendations

Future investigation in this area should include:

- Investigate the use of MACSYMA for determining $\Delta(s)$ from the minimum norm, least squares, and exact square solutions.

- Consider the determination of $\Delta(s)$ as an optimal control problem with path constraints and not just end-point constraints. Thus the controls commanded by $\Delta(s)$ should satisfy the hardware limitations of the control system for reasonable desired system outputs.

- A *systems identification* scheme such as LTC Baker's should provide the determination of the coefficients of $\Delta(s)$ even with the presents of noise in the measurement of the plant matrix $P(s)$. Also, LTC Baker's technique explicitly provides a common denominator for the weighting matrix $\Delta(s)$. [Bak80].

- Consider the role of the weighting matrix as it steers the eigenvectors of P through P_e space. The frequency sensitive weighting matrix provides frequency weighted steering through the input space to the output space. Thus $\Delta(s)$ gives the QFT designer control over which eigenvectors are excited and at which frequency they are excited [Lew88].

- If the QFT designer wishes to use a weighting matrix of constant elements instead of frequency sensitive elements, then Minimum Control Effort analysis might be used to determine weightings for the constant elements δ_{ij}'s [Rei83:387] and [May87].

Appendix A. *MATRIX$_X$ Command Files*

Examples of MATRIX$_X$ command files are presented in this appendix to illustrate how to input a state space model, determine the transfer functions that reside in the original plant matrix $P(s)$ and the failed plants $P_f(s)$, and use the Method of Specified Outputs to numerically determine the weighting matrix $\Delta(s)$ that transforms $P(s)$ into the desired equivalent plant $P_e(s)$. Only skeletons of vital sections of the working command files are included.

```
//MATRIXx Command File
//This program is intended to determine the original plant's transfer
//functions, both healthy and failed, and to determine if the
//healthy and failed plants are controllable
A=[state space model plant matrix]
B=[state space model control matrix]
C=[state space model output matrix]
D=[state space model feedforward matrix]
ns=number of states
l=number of outputs
m=number of inputs
```

program continued

```
//healthy plant matrix of transfer functions
smat=[A B;C D];
[num,den] =tform(smat,ns)
//plant matrix transfer functions with a 50% failure
//of the first control input
Bfail=B;Bfail(:,1)=B(:,1)*.5;sBfail=[A Bfail;C D];
[numBfail,den] =tform(sBfail,ns)
//ask: is the healthy system controllable?
[scon,nscon] =cntrlable(smat,ns)
//ask: is the failed system controllable?
[scon,nsconf] =cntrlable(sBfail,ns)
```

```
//MATRIXx Command File
//This program is intended to determine the numerical value
//of the weighting matrix DELTA(jw) evaluated at specific frequencies.
//DELTAMAT is the matrix whose first m rows is the DELTA(jw) matrix
//evaluated at the first frequency, its second m rows is the DELTA(jw)
//matrix evaluated at the second frequency, ect.
//f=maximum frequency at which DELTA(jw) is evaluated
//m=number of control inputs; for this example, let m=3
//l=number of system outputs; for this example, let l=2
//ns=number of states; for this example, let ns=4
//w=frequency in rad/s
//pd=denominator of DELTA(jw) matrix evaluated at frequency w
//pn(i,j)=numerator of element ij of DELTA(jw) matrix evaluated at w
//patw=evaluation of the lxm plant matrix at frequency w
//pmat=matrix composed of patw matrices evaluated at different w's
//Pemat=desired plant matrix specified by the QFT designer
//for this example, let Pemat=[(.01)/((s+10)**2),0;0,100/((s+10)**2)]
//den=matrix from previous example
//num=matrix from previous example
```

program continued

```
for w=1:f,...
s=jay*w;...
pd=0;
for i=1:(ns+1),...
pd=pd+den(i)*s**(ns-(i-1));...
end;...
pn(1,1)=num(1,4)*s**3+num(1,7)*s**2+num(1,10)*s+num(1,13);...
pn(1,2)=num(1,5)*s**3+num(1,8)*s**2+num(1,11)*s+num(1,14);...
pn(1,3)=num(1,6)*s**3+num(1,9)*s**2+num(1,12)*s+num(1,15);...
pn(2,1)=num(2,4)*s**3+num(2,7)*s**2+num(2,10)*s+num(2,13);...
pn(2,2)=num(2,5)*s**3+num(2,8)*s**2+num(2,11)*s+num(2,14);...
pn(2,3)=num(2,6)*s**3+num(2,9)*s**2+num(2,12)*s+num(2,15);...
for i=1:l,...
for j=1:m,...
patw(i,j)=pn(i,j)/pd;...
end;...
end;...
pmat([(w-1)*l+1:w*l],:)=patw;...
Pemat([(w-1)*l+1:w*l],:)=[(.01)/((s+10)**2), 0; 0, 100/((s+10)**2)];...
DELTAMAT([(w-1)*m+1:w*m],:)=...
patw'*inv(patw*patw')*Pemat([(w-1)*l+1:w*l],:);...
DELTAatW=DELTAMAT([(w-1)*m+1:w*m],:);...
end
```

```
//MATRIXx Command File
//This program is intended to determine the coefficients of
//element ij of the weighting matrix DELTA(jw).
//This program does not place the -1 in the appropriate
//column of the row reduced echelon matrix; the user must
//do that
//deltai=row number of DELTA(jw) element
//deltaj=column number of DELTA(jw) element
//q=degree of numerator polynomial of DELTA(jw) element
//r=degree of denominator polynomial of DELTA(jw) element
```

program continued

```
for w=1:f,...
s=jay*w,...
eldelij(w)=DELTAMAT((w-1)*m+deltai,deltaj);...
LittleN(w,:)=[s**q, s**(q-1), s**(q-2), s**(q-3), s**(q-4), s**(q-5),...
s**(q-6), s**(q-7), s**(q-8), s**(q-9), s**(q-10)];...
BigN(w,:)=LittleN(w,[1:q+1]);...
TinyD(w,:)=[s**r, s**(r-1), s**(r-2), s**(r-3), s**(r-4), s**(r-5),...
s**(r-6), s**(r-7), s**(r-8), s**(r-9), s**(r-10)];...
LittleD(w,:)=TinyD(w,[1:r+1]);...
BigD(w,:)=eldelij(w)*LittleD(w,:);...
end;
BigF=[BigN,-BigD]
sizeBigF=size(BigF),rankBigF=rank(BigF)
ech=rref(BigF)
//The goal of this program is to find any vector of real
//coefficients that lie in the nullspace of the BigF matrix.
//If BigF is not singular then no nullspace exists and the user
//must select higher values of q and (or) r so that BigF is rank
//deficient
//Again, the user must replace the main diagonal 0 with a -1 from any
//column of the row reduced echelon BigF matrix and take the real
//part of that column vector. The first [q+r+2] elements of that
//modified column of the row reduced echelon BigF matrix (cij) lies
//in the nullspace of BigF. As a check, evaluate the product BigF*cij
//to see if the elements of the resulting column vector is almost
//the zero vector (e.g 10E-10 magnitude for each element).
//Once all the transfer functions of the weighting matrix are
//determined, it is advantageous to use MACSYMA to perform the
//matrix multiplication that transforms P(s) into Pe(s).
```

The following is a summary of MATRIX_X built-in functions that the user may find useful. They include CNTRLABLE, FOR LOOPS, INV, JAY, POLY, RANK, REAL, ROOTS, RREF, SIZE, SVD, and TFORM. ,

Appendix B. *Useful MACSYMA Commands*

This appendix provides an example of MACSYMA commands for manipulation of the matrix product $P_e(s) = P(s)\Delta(s)$. Let the original system have 3 inputs and 2 outputs and let the original plant matrix be as given in Eq.(B.1) and let let the weighting matrix be as given in Eq. (B.2).

$$P(s) = \frac{\begin{bmatrix} (s+1) & (s+2) & (s+3) \\ (s+4) & (s+5) & (s+6) \end{bmatrix}}{(s+10)} \tag{B.1}$$

$$\Delta(s) = \frac{\begin{bmatrix} (s+0.1) & (s+0.2) \\ (s+0.3) & (s+0.4) \\ (s+0.5) & (s+0.6) \end{bmatrix}}{(s+0.1)} \tag{B.2}$$

The following MACSYMA commands show how to input the $P(s)$ and $\Delta(s)$ matrices, determine the actual healthy and failed equivalent plant matrices for a 50% loss in control input #1, find the determinant of the actual equivalent plants, and find the zeros of the determinants. In the following, Pn is the matrix of numerator polynomials of $P(s)$, Pd is the common denominator polynomial of $P(s)$, Pnf is the matrix of numerator polynomials of $P(s)$ where control input #1 has lost 50% of available control effectiveness, Dn is the matrix of numerator polynomial of $\Delta(s)$, and Dd is the common denominator polynomial of $\Delta(s)$. Also, Pen is the matrix of healthy equivalent plant numerator polynomials, Ped is the common denominator polynomial of $P_e(s)$, and Penf is the matrix of failed equivalent plant numerator polynomials. Finally, detPen is the determinant of the healthy equivalent plant, detPenf is the determinant of the failed equivalent plant, zdetPen are the zeros of the healthy equivalent plant determinant, and zdetPenf are the zeros of the failed equivalent plant determinant. Recall that $P_e(s) = P(s)\Delta(s)$.

```
Pn:matrix([(s+1),(s+2),(s+3)],[(s+4),(s+5),(s+6)]);
Pd:(s+10);
Pnf[1,1]:0.5*Pn[1,1];
Pnf[2,1]:0.5*Pn[2,1];
Dn:matrix([(s+0.1),(s+0.2),(s+0.3)],[(s+0.4),(s+0.5),(s+0.6)]);
Dd:(s+0.1);
Pen:expand(Pn.Dn);
Ped:expand(Pd*Dd);
Penf:expand(Pnf.Dn);
detPen:expand(determinant(Pen));
detPen:expand(Pen[1,1]*Pen[2,2]-Pen[1,2]*Pen[2,1]);
detPenf:expand(determinant(Penf));
zdetPen:allroots(detPen);
zdetPenf:allroots(detPenf);
```

Other useful MACSYMA commands are writefile(filename), closefile(), save(filename), load(filename), % , and quit().

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Vita

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Use of Quantitative Feedback Theory (QFT) on a multiple-input multiple-output control system requires certain mathematical properties of the plant matrix of transfer functions. In general, the plant matrix P(s) does not posses the necessary or desired mathematical properties for the QFT design to proceed. A frequency sensitive weighting matrix $\Delta(s)$ is used to transform the plant matrix P(s) into the equivalent plant matrix Pe(s) that does satisfy QFT requirements. In matrix notation, the relationship between the equivalent plant, plant, and weighting marix is $Pe(s)=P(s)\,\Delta(s)$. This thesis identifies the necessary and desired characteristics of the equivalent plant Pe(s) for the QFT process, (See reverse side)

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explains the use of the Method of Specified Outputs which generates the frequency sensitive weighting matrix $\Delta(s)$, and calculates several weighting matrices for a 3-input 2-output lateral-directional model of the AFTI/F-16 aircraft. For several control system failures, the mathematical structure of the failed equivalent plant matrices Pef(s) is examined for compliance with QFT requirements. A weighting matrix $\Delta(s)$ is found that produces acceptable equivalent plant matrices for failures of down to 0.01% of available control surface deflections. The use of the software packages MATRIXx and MACSYMA is explained as applied to the Method of Specified Outputs.